C 22714 (Pages : 4) Name.......................................

Reg. No...................................

FOURTH SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2022

(CUCSS)

Mathematics

MT 4E 15—WAVELET THEORY

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

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Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Define translation invariant linear transformation on $l^2(\mathbb{Z}_N)$.

2. Define circulant matrix.

3. State Plancherel's formula in $l^2(\mathbb{Z}_N)$.

4. For $z \in l^2(Z_N)$, show that $(\overline{z})^{\wedge}(m) = \overline{\hat{z}(N-m)}$ for all m.

5. Suppose $z, w \in l^2(\mathbb{Z}_N)$. For $k \in Z$, prove that $z * w(k) = \langle z, R_k \tilde{w} \rangle$, where $\tilde{w}$ is the conjugate reflection of w.

6. Define pth stage wavelet filter sequence.

7. Suppose $M \in \mathbb{N}, N = 2M, z \in l^2(\mathbb{Z}_N)$ and $w \in l^2(\mathbb{Z}_M)$. Then prove that $\langle D(z), w \rangle = \langle z, U(w) \rangle$.

8. Define the Fourier series of a function $f \in L^1([-\pi, \pi))$.

9. Define down sampling operator on $\mathbb{Z}$.

10. Define system matrix of two vectors in $l^2(\mathbb{Z})$.

Turn over

11. Define the notion of Cauchy sequence in $l^2(\mathbb{Z})$.

12. State Cauchy-Schwarz inequality for two functions $f, g \in L^2(\mathbb{R})$.

13. Define the Fourier transform of the function $f \in L^1(\mathbb{R})$.

14. Describe multi-resolution analysis (MRA) with a father wavelet $\varphi \in L^2(\mathbb{R})$.

(14 $\times$ 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question has weightage 2.

15. Let $z = (1, 0, -3, 4) \in l^2(Z_4)$. Find $\hat{z}$, where $\hat{} : l^2(Z_4) \to l^2(Z_4)$ is the discrete Fourier transform.

16. Suppose $N = 2^n, 1 \le p \le n$, and $u_1, v_1, u_2, v_2, \ldots, u_p, v_p$ form a pth stage wavelet filter sequence. Suppose $z \in l^2(Z_N)$. Then prove that the output :

$$\{x_1, x_2, x_3, \ldots, x_p, y_p\}$$

of the analysis phase of the corresponding pth stage wavelet filter bank can be computed using no more than $4N + N \log_2 N$ complex multiplications.

17. Define $T : l^2(Z_4) \to l^2(Z_4)$ by $T(z) = (2z(0) - z(1), iz(1) + 2z(2), z(1), 0)$. Then compute $T(R_1 z)$ and $R_1 T(z)$ where $z = (1, 0, -2, i)$.

18. Suppose $M \in \mathbb{N}$, $N = 2M$, and $w \in l^2(Z_N)$. Then prove that $\{R_{2k} w\}_{k=0}^{M-1}$ is an orthonormal set with M elements if and only if $|\hat{w}(n)|^2 + |\hat{w}(n+M)|^2 = 2$ for $n = 0, 1, \ldots, M-1$.

19. State and prove the folding lemma for vectors in $l^2(Z_N)$.

20. Prove that $L^2([-\pi, \pi)) \subset L^1([-\pi, \pi))$.

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21. Suppose H is a Hilbert space and $T : H \to H$ is a bounded linear transformation. Suppose the series $\sum_{n \in Z} x_n$ converges in H. Then prove that :

$$T\left(\sum_{n \in Z} x_n\right) = \sum_{n \in Z} T(x_n)$$

where the series on the right converges in H.

22. Suppose $w \in l^1(Z)$. Then prove that $\{R_{2k} w\}_{k \in Z}$ is orthonormal if and only if :

$$|\hat{w}(\theta)|^2 + |\hat{w}(\theta + \pi)|^2 = 2 \text{ for all } \theta \in [0, \pi).$$

23. Suppose $f, g \in L^2(\mathbb{R})$ and $x, y \in \mathbb{R}$. Then prove that $\langle R_x f, R_y g \rangle = \langle f, R_{y-x} g \rangle = (f * \tilde{g})(y - x)$.

24. Suppose $f, g \in L^1(\mathbb{R})$. Then prove that $f * g \in L^1(\mathbb{R})$ and $\|f * g\|_1 \le \|f\|_1 \|g\|_1$.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 4.

25. (a) Let $T : l^2(Z_N) \to l^2(Z_N)$ be a translation-invariant linear transformation. Then prove that each element of the Fourier basis F is an eigen vector of T.

(b) Suppose $z \in l^2(Z_N)$ and $k \in Z$. Then prove that for any $m \in Z$,

$$(R_k z)^{\wedge}(m) = e^{-2\pi i m k / N} \hat{z}(m).$$

26. (a) Suppose $M \in \mathbb{N}$ and $N = 2M$. Let $u, v \in l^2(Z_N)$. Then prove that $B = \{R_{2k} v\}_{k=0}^{M-1} \cup \{R_{2k} u\}_{k=0}^{M-1}$ is a first stage wavelet basis for $l^2(Z_N)$ if and only if the system matrix $A(n)$ of u and v is unitary for each $n = 0, 1, .., M - 1$.

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(b) Suppose N is divisible by $2^l, x, y, w \in l^2(Z_{N/2^l})$, and $z \in l^2(Z_N)$. Then prove that :

$U^l(x * y) = U^l(x) * U^l(y)$.

27. (a) Suppose $v, w \in l^1(Z)$ and $z \in l^2(Z)$. Then prove that :

$(z * w)^{\wedge}(\theta) = \hat{z}(\theta)\hat{w}(\theta)$ *a.e.*

(b) Suppose $T : l^2(\mathbb{Z}) \to l^2(\mathbb{Z})$ is a bounded translation-invariant linear transformation. Define $b \in l^2(\mathbb{Z})$ by $b = T(\delta)$. Then show that for all $z \in l^2(\mathbb{Z})$, $T(z) = b * z$.

28. (a) If $f \in L^1(\mathbb{R})$, then prove that $\left|\int_{-\pi}^{\pi} f(x)\, dx\right| \leq \int_{-\pi}^{\pi} |f(x)|\, dx = \|f\|_1$.

(b) Suppose $f \in L^1(\mathbb{R})$ and $\{gt\}_{t>0}$ is an approximate identity. Then prove that for every Lebesgue point x of f, $\lim_{t \to 0^+} g_t * f(x) = f(x)$.

(2 × 4 = 8 weightage)

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C 22713 (Pages : 4) Name.....................................

Reg. No.....................................

FOURTH SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2022

(CUCSS)

Mathematics

MT 4E 14—DIFFERENTIAL GEOMETRY

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

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Part A (Short Answer Questions (1-14).

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Describe the level set at $c = 1$ for the function $f(x_1, x_2) = x_1^2 + x_2^2$.

2. Sketch the vector field on $\mathbb{R}^2$ given by $X(x_1, x_2) = (-x_2, x_1)$ for $(x_1, x_2) \in \mathbb{R}^2$.

3. Show that the n-sphere $x_1^2 + x_2^2 + \ldots + x_n^2 = 1$ is an n-surface.

4. Sketch the cylinder $f^{-1}(0)$, when $f(x_1, x_2) = x_1 - x_2^2$.

5. Find the spherical image of an n-surface $f^{-1}(1)$ where :

   $f(x_1, x_2, \ldots, x_{n+1}) = x_2^2 + x_3^2 + \ldots + x_n^2 - 1$, when $n = 1$.

6. Prove that geodesics have constant speed.

7. True *or* False : "Covariant derivative of a smooth vector field is on the surface is independent of the orientation". Justify your answer.

8. Define Euclidean parallel vectors field along the parametrized curve.

9. Show that $\nabla_{v+w} f = \nabla_v f + \nabla_w f$ for all smooth functions f and $v,\ w \in \mathbb{R}_p^{n+1}$.

10. Find $\nabla_v X$ for $p = (1, 0)$ and $v = (0, 1)$ on the vector field $X(x_1, x_2) = (x_1 x_2, x_2^2)$.

Turn over

11. Find the length of the parametrized curve $\alpha : \mathrm{I} \to \mathbb{R}^2$ given by $\alpha(t) = (t^2, t^3)$ for $t \in [0, 2]$.

12. Let $\kappa_1(p) = 1$ and $\kappa_2(p) = \frac{1}{2}$ be principal curvature of an n-surface S at p. Find the Gaussian Curvature of S at p.

13. Write down the parametrization of the n-plane which passes through $w \in \mathbb{R}^{n+k}$.

14. Let $\phi(\theta, \psi) = (\cos\theta\sin\psi, \sin\theta\sin\psi, \cos\psi)$ for $0 < \theta < 2\pi$ and $0 < \psi < \pi$. Describe ϕ^{-1}.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *from the following ten questions (15-24).*
Each question carries 2 weightage.

15. Let $f : \mathrm{U} \subset \mathbb{R}^n \to \mathbb{R}$ be smooth and $\alpha : \mathrm{I} \to \mathrm{U}$ be a parametrized curve. Show that if $f \circ \alpha$ is constant then α is every where orthogonal to the gradient of f.

16. Define n-surface and given an example of a 1-surface.

17. With the usual notations prove that $(\dot{f}\,\mathrm{X}) = f'\,\mathrm{X} + f\,\dot{\mathrm{X}}$.

18. Let $\alpha : \mathrm{I} \to \mathbb{R}^{n+1}$ be a parametrized curve with $\dot{\alpha}(t) \neq 0$ for all $t \in \mathrm{I}$. Show that there exists a unit speed reparametrization of α.

19. Let X' denotes the covariant derivative of a vector field X along a parametrized curve α. Show that $(\mathrm{X} \cdot \mathrm{Y})' = \mathrm{X}' \cdot \mathrm{Y} + \mathrm{X} \cdot \mathrm{Y}'$.

20. Find the Weingarten map for the circular cylinder $x_2^2 + x_3^2 = a^2$ in $\mathbb{R}^3, (a \neq 0)$.

21. Find the global parametrizations of the plane curve oriented by $\nabla f / \|\nabla f\|$, where $f(x_1, x_2) = \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} - 1, a \neq 0, b \neq 0$.

22. Describe first and second fundamental form of a surface.

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23. Describe the normal section of an n-surface.

24. Let U be an open subset in $\mathbb{R}^n$ and $f : U \to \mathbb{R}$ be a smooth function. Show that $\phi : U \to \mathbb{R}^{n+1}$ defined by $\phi(p) = (p; f(p))$ is a parametrized n-surface in $\mathbb{R}^{n+1}$.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *from the following ten questions (25-28).*
Each question carries 4 weightage.

25. (a) Define integral curve of a vector field.

(b) Let X be a smooth vector field on an open set U in $\mathbb{R}^{n+1}$. Show that there is an integral curve α on X.

(c) Show that for the vector field given by $X(x_1, x_2) = (-x_2, x_1)$, the parametrized curve $\alpha(t) = (\cos t, \sin t)$ is an integral curve for X.

26. (a) Let f, g be smooth functions from an open set U to $\mathbb{R}$. Let $S = f^{-1}(c)\ \nabla f(q) \neq 0$ for all $q \in S$. Let $p \in S$ be an extremal point of g. Show that there exists a real number λ such that $\nabla g(p) = \lambda \nabla f(p)$.

(b) Using Lagrange multiplier fine the extreme point on the unit circle $x_1^2 + x_2^2 = 1$ for the function $g(x_1, x_2) = x_1^2 + x_1 x_2 + x_2^2$.

27. Let $\alpha : I \to \mathbb{R}^3$ be a unit speed parametrized curve in $\mathbb{R}^3$ such that $\dot{\alpha}(t) \times \ddot{\alpha}(t) \neq 0$ for all $t \in I$. Let $T(t) = \dot{\alpha}(t)$, $N(t) = \dfrac{\ddot{\alpha}(t)}{\|\ddot{\alpha}(t)\|}$ and $B(t) = T(t) \times N(t)$ for all $t \in I$. Show that :

(a) $\{N(t), T(t), B(t)\}$ is orthonormal for each $t \in I$.

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(b) There exists a smooth functions $\kappa : I \to \mathbb{R}$ and $t : I \to \mathbb{R}$ such that :

$$\dot{T} = \kappa N$$
$$\dot{N} = -\kappa N + tB$$
$$\dot{B} = -tN.$$

28. (a) Write down the parametrization of torus in $\mathbb{R}^3$.

(b) Find the principal curvatures and Gaussian curvature of the torus.

(2 × 4 = 8 weightage)

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C 22712 (Pages : 3) Name.......................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2022

(CUCSS)

Mathematics

MT 4E 11—GRAPH THEORY

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Each question carries weightage 1.

1. Prove that, if G is simple and bipartite, then $\varepsilon \leq \frac{v^2}{4}$.

2. Prove that in a tree, every two vertices are connected by a unique path.

3. Find a non-trivial simple graph whose automorphism group is the identity.

4. Let G be a connected graph with atleast three vertices. If $e(u, v)$ is a cut-edge in G, then show that either u or v is a cut-vertex.

5. If G is 2-connected, then prove that any two vertices of G lie on a common cycle.

6. Let G be a simple graph and let u and v be non-adjacent vertices in G such that $d(u) + d(v) \geq v$, then prove that G is hamiltonian if and only if G + uv is hamiltonian.

7. Let M be a matching and K be a covering such that $|M| = |K|$, then prove that M is a maximum matching and K is a minimum covering.

8. Let G be a graph with $v - 1$ edges. Then, show that the following statements are equivalent : (a) G is connected ; and (b) G is acyclic.

9. Prove that $\alpha + \beta = \delta$, (with usual notations).

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Turn over

10. Prove that in a bipartite graph G with $\delta > 0$, the number of vertices in a maximum independent set is equal to the number of edges in a minimum edge covering.
11. Prove that in a critical graph, no verted cut is a clique.
12. Show that a graph is planar if and only if each of its blocks is planar.
13. Prove that every sub-graph of a planar graph is planar.
14. Prove that every tournament has a directed Hamilton path.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.
16. If e is a link of G, then prove that $\tau(G) = \tau(G - e) + \tau(G \cdot e)$.
17. Prove that a non-empty connected graph is eulerian if and only if it has no vertices of odd degree.
18. Prove that if G is a simple graph with $\upsilon \geq 3$ and $\delta \geq \frac{\upsilon}{2}$, then G is Hamiltonian.
19. Define closure c(G) of a graph G. Let G be a simple graph with $\upsilon \geq 3$. If c (G) is complete, then prove that G is Hamiltonian.
20. If G is eulerian, then prove that any trail in G constructed by Fleury's algorithm is an Euler tour of G.
21. Prove that graph G has a perfect matching if and only if $o(G - S) \leq |S|$, for all $S \subset V$.
22. If G is bipartite, then prove that $\chi' = \Delta$.
23. If G is k-critical, then prove that $\delta \geq k - 1$.
24. Let υ be a vertex of a planar graph G. Then prove that G can be embedded in the plane in such a way that υ is on the exterior face of the embedding.

(7 × 2 = 14 weightage)

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Part C

Answer any **two** *questions.*
Each question carries weightage 4.

25. For $m < n$, let $f(m, n)$ be the least number of edges that an m-connected graph on n vertices can have. Construct an m-connected graph on n vertices with $f(m, n) = \left\{\frac{mn}{2}\right\}$.

26. If G is a non-Hamiltonian simple graph with $\upsilon \geq 3$, then prove that G is degree majorised by some $C_{m, \upsilon}$. Also prove that if G is bipartite with bipartition (X, Y) where $|X| \neq |Y|$, then G is non-hamiltonian.

27. State Chinese postman problem. Use Fleury's Algorithm to solve Chinese postman problem.

28. Prove that each vertex of a disconnected tournament D with $\upsilon \geq 3$ is contained in a directed k-cycle, $3 \leq k \leq \upsilon$.

(2 × 4 = 8 weightage)

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C 22710 (Pages : 3) Name.......................................

Reg. No...................................

FOURTH SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2022

(CUCSS)

Mathematics

MT 4E 09—FLUID DYNAMICS

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. If ω is the area of the cross-section of a stream filament, prove that the equation of continuity is

$$\frac{\partial}{\partial t}(\rho\omega)+\frac{\partial}{\partial s}(\rho\omega q)=0,$$

where ds is an element of arc of the filament in the direction of flow and q is the speed.

2. Prove that the speed is everywhere the same, then the stream lines are straight.
3. Show that the constancy of circulation in a circuit moving with the fluid in an inviscid fluid in which the density is either constant or is a function of the pressure.
4. Define reducible circuit in a region and give an example of it.
5. Write down the equation of motion in terms of the stream function.
6. Define complex potential and find the speed of the complex potential $w = 2z + 3iz^2$.
7. Find the stagnation points of complex potential $w = \mathrm{U}a\left(\frac{z}{a}\right)^{\frac{\pi}{\alpha}}$.
8. State Circle theorem.
9. What is Cavitation ?
10. Write note on Aerofoil.
11. Find the complex potential due to simple source.

Turn over

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12. Apply Rankine's method to drawing the streamlines for the flow due to two equal sources.

13. Find the image of line source outside the circular cylinder.

14. How are air ship forms formed ?

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Derive the equation of motion of an inviscid fluid.

16. State and prove Kelvin's minimum energy theorem.

17. Derive Bernoulli's equation.

18. A pulse travelling along a fine straight uniform tube filled with gas causes the density at time t and distance x from an origin where the velocity is u_0 to become $\rho_0\phi(Vt-x)$. Prove that the velocity u is given by $V+\frac{(u_0-V)\phi(Vt)}{\phi(Vt-x)}$.

19. Show that the velocity potential $\phi=\frac{1}{2}\log\frac{(x+a)^2+y^2}{(x-a)^2+y^2}$ gives a possible motion, and determine the form of the streamlines.

20. Prove that the velocity potential $\phi=U\left(r+\frac{a^2}{r}\right)\cos\theta$ represents a streaming motion past a fixed circular cylinder.

21. State and prove the theorem of Kutta and Joukowski.

22. Find the stream function of the two-dimensional motion due to two equal sources and an equal sink situated midway between them.

23. OX, OY are fixed rigid boundaries and there is a source at (a, b). Find the form of the streamlines and show that the dividing line is $xy\left(x^2-y^2-a^2+b^2\right)=0$.

24. What arrangement of sources and sinks will give raise to the function $w=\log\left(z-\frac{a^2}{z}\right)$. Draw a rough sketch of the streamlines.

(7 × 2 = 14 weightage)

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Part C

Answer any **two** *questions.*
Each question carries 4 *weightage.*

25. If the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$ prove that the liquid motion is possible and that the velocity is $(\cos\theta)/r^2$. Also determine the streamlines.

26. Show that the Joukowski transformation maps the concentric circles with centre at the origin in the Z-plane into confocal ellipses in the z-plane.

27. Discuss the streaming and circulation for a circular cylinder.

28. In the case of a source at a point A outside a circular disc, prove that the velocity of slip of the fluid in contact with the disc is greatest at the points where the circle is cut by the lines joining A to the ends of the diameter perpendicular to OA, and that its magnitude at these points is

$$\frac{2m.\text{OA}}{\text{OA}^2 - a^2}$$

where is the centre and a the radius of the disc.

(2 × 4 = 8 weightage)

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C 22708 (Pages : 3) Name.......................................

Reg. No...................................

FOURTH SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2022

(CUCSS)

Mathematics

MT 4E 07—ADVANCED FUNCTIONAL ANALYSIS

(2016 and 2018 Admissions)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. Give an example to show that the dual of a separable normed space need not be separable.
2. Let X be a finite dimensional normed space. Prove that x_n converges to x weakly in X if and only if x_n converges to x strongly in X.
3. Show that the dual of a reflexive normed space is reflexive.
4. Give an example to show that the eigenspace of a compact operator corresponding to the eigen value zero can be infinite dimensional.
5. Let H be a Hilbert space and A, B be self-adjoint operators. Then show that A + B is self adjoint.
6. Give an example for a Hilbert- Schmidt operator.
7. Let $A \in BL(H)$ be normal. Show that every spectral value of A is an approximate eigenvalue of A.
8. Define numerical range of a bounded operator on a normed space.
9. Let H be a Hilbert space. If $A, B \in BL(H)$, then show that $\| A^* \| = \| A \|$. Where A^* denotes the adjoint of the bounded operator A.
10. Give an example to show that zero can be the limit point of the spectrum of a compact operator on an infinite dimensional normed space.
11. Illustrate with an example that, in general, weak convergence in a normed space does not imply strong convergence.

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Turn over

12. Let $A \in BL(H)$. Show that $k \in \omega(A)$ if and only if $\bar{k} \in \omega(A^*)$, where $\omega(A)$ represents the numerical range of A.
13. Show that a bounded subset of a Hilbert space is weak bounded.
14. State the generalized Schwarz inequality.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Show that the adjoint T^* of any operator T can be written in the form $T^* = A_1 - iA_2$ where A_1 and A_2 are self adjoint operators.
16. Give an example to show that every linear operator on an infinite dimensional inner product space may not have the adjoint operator.
17. Let X be a separable normed space. Show that every bounded sequence in X' has a weak * convergent subsequence.
18. If T is normal operator on a Hilbert space H, then show that x is an eigen vector of T with eigen value λ if and only if x is an eigen vector of T^* with eigen value $\bar{\lambda}$.
19. Let X be a reflexive normed space. Show that every closed subspace of X is reflexive.
20. Let $A \in BL(H)$ be a Hilbert-Schmidt operator. Show that A* is a Hilbert-Schmidt operator.
21. If T is an arbitrary operator on a Hilbert space H and α and β are scalars such that $|\alpha| = |\beta|$, then show that $\alpha T + \beta T^*$ is normal.
22. Let H be a finite dimensional Hilbert space over $\mathbb{R}$ and $A \in BL(H)$. Suppose that there is an orthonormal basis for H consisting of eigenvectors of A. Show that A is a self adjoint operator.
23. Let $T : C^2 \to C^2$ defined by $Tx = (x_1 + ix_2, x_1 - ix_2)$ for each $x = (x_1, x_2) \in C^2$. Find the adjoint T^*.
24. Let A be a compact operator on $H \neq 0$. Show that every non-zero approximate eigenvalue of A is an eigenvalue of A.

(7 × 2 = 14 weightage)

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Part C

Answer any **two** *questions.*
Each question carries weightage 4.

25. Let $1 \le p \le \infty$ and $1/p + 1/q = 1$. Show that the dual of c_o with the norm $\| \|_\infty$ is linearly isometric to l^1.

26. Let X be a Banach space which is uniformly convex in some equivalent norm. Show that X is reflexive.

27. State and prove the Riesz representation theorem.

28. Let H be a Hilbert space and $A \in BL(H)$. Show that there is a unique $B \in BL(H)$ such that for all $x, y \in H, \langle A(x), y \rangle = \langle x, B(y) \rangle$.

(2 × 4 = 8 weightage)

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C 22706 (Pages : 3) Name....................................

Reg. No....................................

FOURTH SEMESTER M.Sc. DEGREE [SUPPLEMENTARY] EXAMINATION APRIL 2022

(CUCSS)

Mathematics

MT 4E 02—ALGEBRAIC NUMBER THEORY

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

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Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Find the order of the group G/H where G is free abelian with Z- basis x, y, z and H generated by $2x, 3y, 7z.$

2. Find all submodules of $\mathbb{Z}$.

3. Is $\sqrt{1+\sqrt{2}}+\sqrt{1-\sqrt{2}}$ an algebraic number ? An algebraic integer ? Give reasons.

4. Define a unique factorization domain. Is $\mathbb{Q}$ a U F D ?

5. Prove that 3 has no proper factors in $\mathbb{Z}\left[\sqrt{-5}\right]$.

6. Let $\mathcal{O}$ be the ring of integers in a number field K and let $x, y \in \mathcal{O}$. Prove that x is a unit if and only if $N(x) = \pm 1$.

7. Define a Euclidean domain. Give an example.

8. Define the norm of an ideal.

9. Find : integral basis and discriminant for $\mathbb{Q}\left[\sqrt{-11}\right]$.

Turn over

10. Let $K = \mathbb{Q}(\zeta)$ where $\zeta = e^{\frac{2\pi i}{5}}$. Find $N_K(\zeta^2)$ and $N_K(\zeta^2)$.

11. In $\mathbb{Z}[\sqrt{-5}]$ show that $\sqrt{-5}$ divides $a + b\sqrt{-5}$ if and only if 5 divides a.

12. Show that subring of a Noetherian ring need not be Noetherian.

13. Find the units in the set of Gaussian integers.

14. Define a regular prime. Give an example.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question has weightage 2.

15. Prove that two different bases of a free abelian group have the same number of elements.

16. Prove that the set of algebraic integers forms a subring of the set of algebraic numbers.

17. Find integral basis and discriminant of $\mathbb{Q}(\sqrt{3}, \sqrt{5})$.

18. Prove that the ring of integers $\mathcal{O}$ in a number field K is Noetherian.

19. Prove that the ideal $\langle 2, 1 + \sqrt{-5} \rangle$ not a principal ideal in $\mathbb{Z}[\sqrt{-5}]$.

20. Prove that if $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ is a basis for K over $\mathbb{Q}$ then $\{\sigma(\alpha_1), \sigma(\alpha_2), ..., \sigma(\alpha_n)\}$ is a linearly independent set over $\mathbb{R}$.

21. Let K be a number field of degree $n = s + 2t$. Prove that every non-zero ideal I of $\mathcal{O}$ equivalent to an ideal whose norm is $\leq \left(\frac{2}{\pi}\right)^t \sqrt{|\Delta|}$.

22. The discriminant of $\mathbb{Q}(\zeta)$ where $\zeta = e^{\frac{2\pi i}{p}}$ and p is an odd prime is $(-1)^{\frac{p-1}{2}} \cdot p^{p-2}$.

23. Prove that the only roots of unity in $K = \mathbb{Q}(\zeta)$ are $\pm \zeta^s$ for integers s.

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24. If $p(t) \in \mathbb{Z}[t]$ is a monic polynomial all of whose zeros in $\mathbb{C}$ have absolute value 1, then prove that every zero is a root of unity.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 4.

25. (a) If K is a number field then prove that $K = \mathbb{Q}(\theta)$ for some algebraic number θ.

(b) Prove : For a number field K, if $\alpha \in K$, then for some non-zero $c \in \mathbb{Z}$, $c\alpha \in \mathcal{O}$.

26. (a) Find the ring of integers of $\mathbb{Q}\left(\sqrt{2}, i\right)$.

(b) Let $K = \mathbb{Q}\left(\sqrt[4]{2}\right)$. Find all monomorphisms of $\mathbb{Q}\left(\sqrt[4]{2}\right) \to \mathbb{C}$, minimum polynomial over $\mathbb{Q}$ and field polynomial over K.

27. (a) Derive all solutions for the Fermat's equation $x^n + y^n = z^n$ for $n = 2$.

(b) Show that if π is an irreducible in $\mathbb{Z}[i]$ then $\mathbb{Z}[i]/\langle\pi\rangle$ is a field.

28. (a) Prove that every number field possesses an integral basis and the additive group of $\mathcal{O}$ is free abelian of rank n equal to the degree of K.

(b) Prove that an algebraic number α is an algebraic integer if and only if the minimum polynomial over $\mathbb{Q}$ has co-efficients in $\mathbb{Z}$.

(2 × 4 = 8 weightage)

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C 22705 (Pages : 3) Name.....................................

Reg. No.................................

FOURTH SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION, APRIL 2022

(CUCSS)

Mathematics

MT4E01—COMMUTATIVE ALGEBRA

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Prove that the nilpotent elements in a commutative ring with unity is a zero divisor (unless A = 0).
2. Give an example of a ring with exactly one maximal ideal.
3. Define a local ring. Give an example
4. State Nakayama's Lemma.
5. Define a Flat module.
6. Prove that the operation S^{-1} commutes with formation of finite products.
7. Write the primary ideals in $\mathbb{Z}$.
8. Define a Noetherian ring.
9. Define a composition series.
10. State Structure Theorem for Artin rings.
11. State Second Uniqueness Theorem.
12. Prove that $\mathfrak{a} \cap \mathfrak{b} = \mathfrak{a}\mathfrak{b}$ provided $\mathfrak{a} + \mathfrak{b} = (1)$
13. Define radical of an ideal in a ring.
14. All principal ideal Domains are Noetherian. Prove or disprove.

(14 × 1 = 14 weightage)

Turn over

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Part B

Answer any **seven** *questions.*
Each question has weightage 2.

15. Let $\mathfrak{a}$ and $\mathfrak{b}$ ideals in a ring A such that $r(\mathfrak{a})$ and $r(\mathfrak{b})$ are coprime. Prove that $\mathfrak{a}$ and $\mathfrak{b}$ are coprime.

16. Prove that a local ring contains no idempotent $\neq 0, 1$.

17. Let A be a local ring, $\mathfrak{m}$ its maximal ideal and M be a finitely generated A-module. Let x_i $(1 \leq i \leq n)$ be elements of M whose images in $M/\mathfrak{m}M$ form a basis for this vector space. Prove that x_i generate M.

18. Let A be a commutative ring with unity and M be an A-module. Explain the construction of $S^{-1}M$.

19. If $\mathfrak{N}$ is the nilradical of A, then prove that the nilradical of $S^{-1}A$ is $S^{-1}\mathfrak{N}$.

20. Let $\mathfrak{q}$ be a primary ideal in a ring A and x be an element of A. If $x \notin \mathfrak{q}$ then prove that $(\mathfrak{q} : x)$ is $\mathfrak{p}$ -primary.

21. Let B be an integral domain and K its field of fractions. If B is a valuation ring of K then prove that B is a local ring.

22. For a k-vector space V, prove that $a.c.c. \Rightarrow$ finite dimension.

23. If A is Noetherian and ϕ is a homomorphism of A onto a ring B then prove that B is Noetherian.

24. Prove that a ring A is Artin $\Leftrightarrow$ A is Noetherian and dim.A = 0.

(7x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 4.

25. (a) Prove that the set $\mathfrak{N}$ of nilpotent elements in a ring A is an ideal and $A/\mathfrak{N}$ has no nilpotent element $\neq 0$.

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(b) Let A be a ring and $\mathfrak{R}$ be the Jacobson radical of A. Prove that $x \in \mathfrak{R} \Leftrightarrow 1 - xy$ is a unit in A for all $y \in A$.

26. (a) $A \subseteq B$ be integral domains and B integral over A. Prove that B is a field if and only if A is a field.

(b) State and prove Going-down Theorem.

27. (a) If A is Noetherian then prove that A $[x_1, x_2, \ldots x_n]$ is also Noetherian.

(b) Prove that an Artin ring has only a finite number of maximal ideals.

28. (a) Let k be a field and E be a finitely generated k-algebra. If E is a field then prove that it is a finite algebraic extension of k.

(b) Let A be a local ring and m be its maximal ideal. Prove that $\dim_k (m/m^2) \leq 1 \Rightarrow$ every ideal in A is principal (where $k = A/m$).

(2 × 4 = 8 weightage)

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C 22573–A (Pages : 5) Name..................................

Reg. No...............................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

April 2021 Session for SDE/Private Students

(CBCSS)

Mathematics

MTH 4E 13—WAVELET THEORY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

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INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH 4E 13—WAVELET THEORY

(Multiple Choice Questions for SDE Candidates)

1. If $\wedge : l^2(\mathbb{Z}_N) \to l^2(\mathbb{Z}_N)$ is the discrete Fourier transform, then ————.

   (A) $\hat{z}(m+N) = \hat{z}(m)$ for $m \in \mathbb{Z}$.

   (B) $\hat{z}(m+N) = \hat{z}(2m)$ for $m \in \mathbb{Z}$.

   (C) $\hat{z}(m+N) = \hat{z}(3m)$ for $m \in \mathbb{Z}$.

   (D) None of the above.

2. For $z \in l(\mathbb{Z}_N), \|z\|^2 =$ ————.

   (A) $\|\hat{z}\|^2$.

   (B) $\frac{1}{N}\|\hat{z}\|^2$.

   (C) $\frac{2}{N}\|\hat{z}\|^2$.

   (D) None of the above.

3. If $F = \{F_0, F_1, \ldots, F_{N-1}\}$ is the Fourier basis for $l^2(\mathbb{Z}_N)$, then $z =$ ————.

   (A) $\sum_{m=0}^{N/2-1} \hat{z}(m) F_m$.

   (B) $\sum_{m=0}^{N-1} \hat{z}(m) F_m$.

   (C) $\sum_{m=0}^{N-1} \hat{z}(m/2) F_m$.

   (D) None of the above.

4. If $F = \{F_0, F_1, \ldots, F_{N-1}\}$ is the Fourier basis for $l^2(\mathbb{Z}_N)$, then $\hat{z} =$ ————.

   (A) $[z]_F$.

   (B) $2[z]_F$.

   (C) $\frac{1}{2}[z]_F$.

   (D) None of the above.

5. $W_4 =$ ————.

   (A) $W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$.

   (B) $W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$.

   (C) $W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & 1 & 1 \end{bmatrix}$.

   (D) $W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ i & i & -1 & -i \end{bmatrix}$.

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6. $W_4^{-1} = $ ———.

(A) $\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$.

(B) $\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$.

(C) $\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & i \end{bmatrix}$.

(D) None of the above.

7. Suppose $N = 6$, $k = 2$ and $z = (2, 3 - i, 2i, 4 + i, 0, 1)$, then $(R_2 z)(0) = $ ———.

(A) 0.

(B) 1.

(C) 2.

(D) $3 - i$.

8. Suppose $N = 6$, $k = 2$, and $z = (7, 3 - i, 2i, 4 + i, 7, 1)$, then $(R_2 z)(0) = $ ———.

(A) 0.

(B) 1.

(C) 2.

(D) None of the above options.

9. Suppose $N = 6$, $k = 3$, and $z = (2, 3 - i, 2i, 4 + i, 5, 1)$, then $(R_3 z)(0) = $ ———.

(A) 2.

(B) $3 - i$.

(C) 5.

(D) $4 + i$.

10. Suppose $N = 6$, $k = 4$, and $z = (2, 3 - i, 2i, 4 + i, 5, 1)$, then $(R_4 z)(0) = $ ———.

(A) 2.

(B) $3 - i$.

(C) 5.

(D) $2i$.

11. Suppose $N = 6$, $k = 4$, and $z = (2, 3 - i, 2i, 4 + i, 5, 1)$, then $(R_4 z)(1) = $ ———.

(A) 2.

(B) $3 - i$.

(C) 5.

(D) $4 + i$.

12. Suppose $N = 6$, $k = 2$, and $z = (2, 3 - i, 2i, 4 + i, 5, 1)$ then $(R_2 z)(4) = $ ———.

(A) 2

(B) $2i$.

(C) 5.

(D) $2i$.

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13. Suppose N = 6, k = 4, and $z = (2, 3 - i, 2i, 4 + i, 5, 1)$ then $(R_4 z)(4) =$ ———.

(A) 2
(B) $3 - i$.
(C) 5.
(D) $2i$.

14. The vector $(z * w)^\wedge \in l^2(Z_4)$ where

$$\hat{z} = (4, 1 + i, -2, 1 - i)$$

and

$$\hat{w} = (1 + 2i, -2 + i, 1, i)$$

is ———.

(A) $(4 + 8i, -3 - i, -2, 1 + i)$.
(B) $(4 - 8i, -3 - i, -2, 1 + i)$.
(C) $(4 + 8i, -3 - i, 2, 1 + i)$.
(D) $(4 + 8i, 3 + i, -2, 1 + i)$.

15. Let $z, w \in l^2(\mathbb{Z}_N)$. Then for any $k, j \in \mathbb{Z}$, $\langle R_k z, R_j w \rangle =$ ———.

(A) $\langle R_{k+j} z, w \rangle$.
(B) $\langle R_{k-j} z, w \rangle$.
(C) $\langle R_{k-2j} z, w \rangle$.
(D) $\langle R_{k+2j} z, w \rangle$.

16. For a sequence $z = (z(n))_{n \in \mathbb{Z}}$, the upsampling operator on $l^2(\mathbb{Z})$ is defined by ———.

(A) $U(z)(n) = \begin{cases} z(n/2) & \text{if } n \text{ is even,} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$

(B) $U(z)(n) = \begin{cases} z(n/2) & \text{if } n \text{ is odd,} \\ 0 & \text{if } n \text{ is even.} \end{cases}$

(C) $U(z)(n) = \begin{cases} z(2n) & \text{if } n \text{ is even,} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$

(D) $U(z)(n) = \begin{cases} z(2n) & \text{if } n \text{ is odd,} \\ 0 & \text{if } n \text{ is even.} \end{cases}$

17. Suppose $u, v \in l^2(\mathbb{Z})$. The system matrix of u and v is A $(\theta) =$ ———.

(A) $\sqrt{2}\begin{bmatrix} \hat{u}(\theta) & \hat{v}(\theta) \\ \hat{u}(\theta + \pi) & \hat{v}(\theta + \pi) \end{bmatrix}$.

(B) $\begin{bmatrix} \hat{u}(\theta) & \hat{v}(\theta) \\ \hat{u}(\theta + \pi) & \hat{v}(\theta + \pi) \end{bmatrix}$.

(C) $\frac{1}{2}\begin{bmatrix} \hat{u}(\theta) & \hat{v}(\theta) \\ \hat{u}(\theta + \pi) & \hat{v}(\theta + \pi) \end{bmatrix}$.

(D) $\frac{1}{\sqrt{2}}\begin{bmatrix} \hat{u}(\theta) & \hat{v}(\theta) \\ \hat{u}(\theta + \pi) & \hat{v}(\theta + \pi) \end{bmatrix}$.

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18. Suppose $N = 2^n$ for some $n \in \mathbb{N}$. Then $\#_N \le$ ————.

(A) $\frac{1}{3} N \log_2 N$.

(B) $\frac{1}{4} N \log_2 N$.

(C) $\frac{1}{2} N \log_2 N$.

(D) None of the above.

19. Suppose H is a Hilbert space, $\{a_j\}_{j \in Z}$ is an orthonormal set in H, and $f \in H$. Then ————.

(A) $\sum_{j \in Z} |\langle f, a_j \rangle|^2 \ge \|f\|^2$.

(B) $\sum_{j \in Z} |\langle f, a_j \rangle|^2 \le \|f\|^2$.

(C) $\sum_{j \in Z} |\langle f, a_j \rangle|^2 = \|f\|^2$.

(D) $\sum_{j \in Z} |\langle f, a_j \rangle|^2 \le 4\|f\|^2$.

20. Let $T : l^2(\mathbb{Z}_N) \to l^2(\mathbb{Z}_N)$ be a translation-invariant linear transformation. Then ————.

(A) det T = 1.

(B) det T = 0.

(C) det T = ± 1.

(D) None of the above.

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D 22573 (Pages : 4) Name.................................

Reg. No.................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

April 2021 Session for SDE/Private Students

(CBCSS)

Mathematics

MTH 4E 13—WAVELET THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

Covid Instructions are not applicable for Pvt/SDE students (April 2021 session)

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section/Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section/sub part/sub division may be ignored.*
4. *There will be an overall ceiling for each Section/Part that is equivalent to the maximum weightage of the Section/Part.*

Part A

Answer **all** *questions.*
Each question carries a weightage of 1.

1. Examine whether the matrix $\begin{bmatrix} 3 & 2+i & -1 & 4i \\ 4i & 3 & 2+i & -1 \\ -1 & 4i & 3 & 2+i \\ 2+i & -1 & 4i & 3 \end{bmatrix}$ is circulant or not.
2. Suppose $z,\ w \in l^2(\mathbb{Z}_N)$. For $k \in Z$, prove that $z * \overline{w}(k) = \langle z, R_k w \rangle$.
3. If $X : Z_N \to C$ is mutiplicative, and if $X \neq 0$ then prove that $X(n) = [X(1)]^n$.
4. Define upsampling operator on $\mathbb{Z}$.
5. Define homogeneous wavelet system for $l^2(\mathbb{Z})$.

Turn over

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6. Give an inner product of $L^2(\mathbb{R})$.

7. Define translation and conjugate reflection of a function $f:\mathbb{R}\to\mathbb{C}$.

8. Define wavelet system for $L^2(\mathbb{R})$.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each module.*
Each question carries a weightage of 2.

Module I

9. Let $w=(2,4,+4i-6,4-4i)\in l^2(\mathbb{Z}_4)$ and it is given that $W_4=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -i & -1 & i\\ 1 & -1 & 1 & -1\\ 1 & i & -1 & -i\end{bmatrix}$. Find $\tilde{w}$.

10. Suppose $z,w\in l^2(\mathbb{Z}_N)$. Then prove that for each m, $(\widehat{z*w})(m)=\overline{z}(m)\hat{w}(m)$.

11. Let $z,w\in l^2(\mathbb{Z}_N)$. Then prove that $\langle R_k z, R_j w\rangle=\langle R_{k-j}z, w\rangle$ for any $k, \in\mathbb{Z}$.

Module II

12. If f and g are members of $L^2([-\pi,\pi))$, then prove that $f\cdot g\in L^1([-\pi,\pi))$.

13. Suppose H is a Hilbert space and $T: H\to H$ is a bounded linear transformation. Suppose the series $\sum_{n\in Z}x_n$ converges in H. Then prove that $T(\sum_{n\in Z}x_n)=\sum_{n\in Z}T(x_n)$ where the series on the right converges in H.

14. Suppose H is a Hilbert space, $\{a_j\}_{j\in Z}$ is an orthonormal set in H, and $f\in H$. Then prove that the sequence $\{\langle f,a_j\rangle\}_{j\in Z}$ belongs to $l^2(\mathbb{Z})$ with $\sum_{j\in Z}\left|\langle f,a_j\rangle\right|^2\le\|f\|^2$.

Module III

15. By means of examples, show that there is no containment between $L^1(\mathbb{R})$ and $L^2(\mathbb{R})$.

16. Suppose $f,g\in L^1(\mathbb{R})$. Then prove that $f*g\in L^1(\mathbb{R})$ and $\|f*g\|_1\le\|f\|_1\|g\|_1$.

17. Suppose $f,g\in L^1(\mathbb{R})$ and $\hat{f},\hat{g}\in L^1(\mathbb{R})$. Then prove that $\langle\hat{f},\hat{g}\rangle=2\pi\langle f,g\rangle$.

(6 × 2 = 12 weightage)

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Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. (a) Let $T: l^2(Z_N) \to l^2(Z_N)$ be a translation-invariant linear transformation. Then prove that each element of the Fourier basis F is an eigen vector of T.

    (b) Define $T: l^2(Z_4) \to l^2(Z_4)$ by $T(z)(n) = z(n) + 2z(n+1) + z(n+3)$. Find the eigen values and eigen vectors of T, and diagonalize the matrix A representing T in the standard basis.

19. (a) Suppose $N = 2^n$ for some $n \in \mathbb{N}$. Then prove that $\#_N \le \frac{1}{2} N \log_2 N$, where $\#_N$ is the least number of complex multiplications required to compute the D.F.T.

    (b) Suppose $M \in \mathbb{N}$, $N = 2M$, and $w \in l^2(Z_N)$. Then prove that $\{R_{2k} w\}_{k=0}^{M-1}$ is an orthonormal set with M elements if and only if

$$|\hat{w}(n)|^2 + |\hat{w}(n+M)|^2 = 2 \text{ for } n = 0, 1, \ldots, M-1.$$

20. (a) Suppose $f \in L^1([-\pi, \pi))$ and $\langle f, e^{in\theta} \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) e^{-in\theta} d\theta = 0$ for all $n \in Z$. Then prove that $f(\theta) = 0$ a.e.

    (b) Suppose $\theta_0 \in (-\pi, \pi)$ and $\alpha > 0$ is sufficiently small that $-\pi < \theta_0 - \alpha < \theta_0 + \alpha < \pi$. Define intervals.

$$I = (\theta_0 - \alpha, \theta_0 + \alpha),$$

and

$$J = (\theta_0 - \alpha/2, \theta_0 + \alpha/2).$$

Then prove that there exists $\delta > 0$ and a sequence of real-valued trigonometric polynomials $\{p_n(\theta)\}_{n=1}^{\infty}$ such that

(i) $p_n(\theta) \ge 1$ for $\theta \in I$.

(ii) $p_n(\theta) \ge (1+\delta)^n$ for $\theta \in J$.

(iii) $|p_n(\theta)| \le 1$ for $\theta \in [-\pi, \pi) \setminus I$.

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21. (a) Suppose $f, g \in L^2(\mathbb{R})$. Then prove that for all $x \in \mathbb{R}$ $|(f * g)(x)| \leq \|f\| \|g\|$.

(b) Suppose $f \in L^2(\mathbb{R})$. Then prove that

$$f = \left(\hat{f}\right)^{\vee}$$

and

$$f = \left(\check{f}\right)^{\wedge}.$$

(2 × 5 = 10 weightage)

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C 22572-A (Pages : 4) Name..................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE [REGULAR/SUPPLEMENTARY] EXAMINATION, APRIL 2022

April 2021 Session for SDE/Private Students

(CBCSS)

Mathematics

MTH4E12—REPRESENTATION THEORY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	Total No. of Questions : 20	Maximum : 5 Weightage

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

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MTH4E12—REPRESENTATION THEORY

(Multiple Choice Questions for SDE Candidates)

1. Let A (x) be a representation of a group G. Which of the following is false ?

   (A) $A(x^{-1}) = (A(x))^{-1}$ for every $x \in G$.
   (B) A (1) = I.
   (C) Ker $A = \{1\}$.
   (D) $A(x)A(y) = A(xy)$ for every $x, y \in G$.

2. A function having the same value throughout a conjugacy class is called :

   (A) Constant function.
   (B) Group function.
   (C) Conjugate function.
   (D) Class function.

3. If χ and $\bar{\chi}$ are the characters of A (x) and $A^{+}(x)$ respectively, then :

   (A) χ is simple if and only if $\bar{\chi}$ is simple.
   (B) χ is simple but $\bar{\chi}$ need not be simple.
   (C) $\bar{\chi}$ is simple even when χ is not simple.
   (D) None of these.

4. Suppose that U and V are irreducible G—modules over K. Then a G—homomorphism $\theta : V \to U$ is :

   (A) Always an isomorphism.
   (B) Always the zero map.
   (C) Neither zero nor an isomorphism.
   (D) Either zero or an isomorphism.

5. The character of an irreducible representation is called :

   (A) Trivial character.
   (B) Compound character.
   (C) Simple character.
   (D) None of these.

6. Let $\chi^{(1)}, \chi^{(2)}, \ldots, \chi^{(r)}$ be the complete set of simple characters of a group G of order g corresponding to the irreducible representations $F^{(1)}, F^{(2)}, \ldots, F^{(r)}$. If A is an arbitrary representation, and ϕ is the character of A, then the Fourier analysis of ϕ or A is :

   (A) $\phi = \sum_{j=1}^{g} d_j \chi^{(j)}$ where $d_j = \langle \phi, \chi^{(j)} \rangle$.
   (B) $\phi = \sum_{j=1}^{g} d_j \chi^{(j)}$ where $d_j = \langle \phi, \phi \rangle$.
   (C) $\phi = \sum_{j=1}^{r} d_j \chi^{(j)}$ where $d_j = \langle \phi, \phi \rangle$.
   (D) $\phi = \sum_{j=1}^{r} d_j \chi^{(j)}$ where $d_j = \langle \phi, \chi^{(j)} \rangle$.

7. Let R (x) = $r_{ij}(x)$ be the right regular representation of a G-module $G_{\mathbb{C}}$ with basis vectors $[x_1, x_2, .., x_g]$. If $x_i x_j = x_{\mu(i,j)}$ in G, then $r_{ij}(x_s) =$

   (A) $\delta_{\mu(j,s),i}$.
   (B) $\delta_{\mu(j,s),j}$.
   (C) $\delta_{\mu(i,s),j}$.
   (D) $\delta_{\mu(i,s),i}$.

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