C 2744 (Pages : 3) Name.....................................

Reg. No...............................

FOURTH SEMESTER P.G. DEGREE EXAMINATION, APRIL 2021

(CCSS)

Statistics

STA 4E 04—OPERATION RESEARCH–II

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Use of Calculator is permitted.

Part I

Answer any **four** *questions.*
Each question carries 4 marks.

I. (a) Define quadratic programming problem. How does a quadratic programming problem differ from a linear programming problem ?

(b) Solve the following linear programming problem using the method of Lagrangian's multipliers :

Minimize $f(x_1, x_2, x_3) = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$

subject to the constraints :

$x_1 + x_2 + x_3 = 11$ and $x_1, x_2, x_3 \geq 0$.

(c) Define dynamic programming problem and state Bellman's principle of optimality.

(d) Define stage, slack variable, decision variable, return function and optimum return in dynamic programming problem.

(e) Define Inventory System. Distinguish between continuous review and periodic review inventory systems.

(f) Explain newsboy problem.

(g) A firm is considering replacement of a machine, whose cost price is Rs. 12,200 and the scrap value Rs. 200. The running costs are found from experience to be as follows :

Year	1	2	3	4	5	6	7	8
Running cost in rupees	200	500	800	1200	1800	2500	3200	4000

Determine the optimum replacement policy.

(h) Describe the methodology of simulation.

(4 × 4 = 16 marks)

Turn over

Part II

Answer **either** *part (A)* **or** *part (B) of all questions.*
Each question carries 16 *marks.*

II. (A) (i) Define geometric programming problem. Explain its importance.

(ii) Find optimum solution to the following problem by geometric programming method :

Minimize $\left(x_1, x_2, x_3\right) = \frac{7x_1}{x_2} + \frac{3x_2}{x_3^2} + \frac{5x_2x_3}{x_1^3} + x_1x_2x_3$ and $x_1, x_2, x_3 \geq 0$.

(5 + 11 = 16 marks)

Or

(B) (i) Develop optimal decision policy and explain general procedure for obtaining optimum solution of problem using the dynamic programming approach.

(ii) Solve the following problem by dynamic programming method :

Minimize $f\left(x_1, x_2, x_3\right) = x_1^2 + x_2^2 + x_3^2$

subject to $x_1 + x_2 + x_3 = 10$ and $x_1, x_2, x_3 \geq 0$.

(8 + 8 = 16 marks)

III. (A) (i) Describe Wolfe's modified simplex method of solving quadratic programming problem.

(ii) Find optimum solution to the following QPP by Wolfe's method :

Maximize Z = $2x + y - x^2$

subject to the constraints :

$2x + 3y \leq 6, 2x + y \leq 4$ and $x, y \geq 0$.

(7 + 9 = 16 marks)

Or

(B) (i) Define general non-linear programming problem. Derive the Kuhn-Tucker conditions.

(ii) Find optimum solution of the problem given below using the Kuhn-Tucker conditions.

Minimize $f\left(x_1, x_2\right) = \left(x_1 + 1\right)^2 + \left(x_2 - 2\right)^2$

subject to $0 \leq x_1 \leq 2$ and $0 \leq x_2 \leq 1$.

(8 + 8 = 16 marks)

IV. (A) (i) Distinguish between deterministic and stochastic inventory models.

(ii) Stating the assumptions, derive the classic EOQ formula.

(iii) The demand for a particular item is 18000 units per year. The holding cost per unit is Rs.1.20/- per year and the procurement cost is Rs.400/- per unit. Shortages are not allowed and instantaneous replacement rate. Determine optimum order quantity and optimum time between orders.

(iv) Write short notes on production lot size inventory model.

(3 + 6 + 4 + 3 = 16 marks)

Or

(B) (i) Stating assumptions, develop and find optimum solution of a continuous review inventory model where demand during lead time is backlogged and uncertain.

(ii) The demand rate of manufacturing items of a company is 1000 per month. It costs Rs. 100 to place an order for a new shipment. The holding cost per item per month is Rs. 2 and shortage cost per item is Rs. 10. The demand during lead time is backlogged and uncertain, distributed as uniformly over the range (0, 100). Determine the optimal inventory policy.

(8 + 8 = 16 marks)

V. (A) (i) Develop a model for optimum replacement of equipment whose maintenance costs increase with time and value of money changes with time.

(ii) The initial cost of an item is Rs. 15,000 and running costs for different periods are given below :

Year	1	2	3	4	5	6	7
Running costs (in rupees) :	2,500	3,000	4,000	5,000	6,500	8,000	10,000

What is the optimum replacement policy if the capital is worth 10% and there is no salvage value ?

(8 + 8 = 16 marks)

Or

(B) (i) What is simulation ? Why simulation is used ? What are different types of simulation models ? Explain Monte-Carlo simulation technique.

(ii) Write short notes on the following :

(a) Convolution method of generating random samples.

(b) Group replacement policy.

(10 + 6 = 16 marks)

[4 × 16 = 64 marks]

C 2743 (Pages : 3) Name.......................................

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FOURTH SEMESTER P.G. DEGREE EXAMINATION, APRIL 2021

(CCSS)

Statistics

STA 4E 03—LIFE TIME DATA ANALYSIS

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (a) Distinguish between type I and type II censoring. Give examples for each.

(b) Write a short note on mixture models.

(c) Write down the expressions for finding Nelson-Aalen estimate and its variance. What is its use ?

(d) What is the difference between plot of survivor function and P-P plot ?

(e) The first eight observations in a random sample of 12 lifetimes in hours, from an assumed exponential distribution are in hours 31, 58, 157, 185, 300, 470, 497, 673. Find the MLE of θ and 95 % confidence interval for θ.

(f) Write a short note on Wald type confidence procedures.

(g) Why are log location scale regression models called accelerated failure time models ?

(h) Let $t_{(r)}$ be the r^{th} smallest observation in a random sample of size n from exponential distribution with mean θ. Prove that $E\left(t_{(r)}\right) = \theta \sum_{i=1}^{r} \frac{1}{n-i+1}$.

(4 × 4 = 16 marks)

Turn over

Section B

Answer **either** *Part (A)* **or** *Part (B) of* **all** *questions.*
Each question carries 16 *marks.*

II. (A) a) Discuss the monotonicity of hazard function of lognormal distribution.

b) Derive the expression for the survivor function of log location scale model.

(9 + 7 = 16 marks)

Or

(B) a) Describe the general formulation of right censoring.

b) Derive the relation between hazard rate and mean residual life function.

(10 + 6 = 16 marks)

III. (A) a) Define empirical survivor function and Kaplan-Meier estimate of survivor function. Show that if all the observations are uncensored, then Kaplan-Meier estimate becomes empirical survivor function.

b) Derive Kaplan-Meier estimate as a nonparametric MLE of survivor function.

(8 + 8 = 16 marks)

Or

(B) a) Define left truncated data. Explain the methods for estimating survivor function of a left truncated data.

b) Discuss standard life table methods.

(8 + 8 = 16 marks)

IV. (A) a) Obtain the likelihood ratio test procedures for comparing two exponential distributions based on independent samples.

b) Develop likelihood based methods for location-scale distributions under censored samples.

(8 + 8 = 16 marks)

Or

(B) a) Obtain the exact methods for type II censored test plans based on exponential distribution.

b) Describe the maximum likelihood inference procedures for gamma distribution, given an uncensored data.

(8 + 8 = 16 marks)

V. (A) a) Discuss the likelihood based inference procedures for Accelerated failure time models.

b) Explain how regression models can be used for comparing two or more distributions.

(8 + 8 = 16 marks)

Or

(B) a) Justify Cox likelihood as a partial likelihood.

b) Write a short note on multivariate life time distributions.

(9 + 7 = 16 marks)

[4 × 16 = 64 marks]

C 2742 (Pages : 3) Name.......................................

Reg. No...................................

FOURTH SEMESTER P.G. DEGREE EXAMINATION, APRIL 2021

(CCSS)

Statistics

STA 4E 04—OPERATION RESEARCH—II

(2010 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Part A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (a) Give the sufficient condition for a NLPP with multiple constraints.

(b) Give the Wolfs algorithm for solving QPP.

(c) What do you mean by minimum path problem ? Give its significant.

(d) Explain the practical aspects of Dynamic Programming Problem.

(e) What is a newsboy problem ?

(f) Derive the expected number of back orders when the lead time distribution is normal.

(g) What do you mean by simulation ? Explain acceptance-rejection method.

(h) State some simple replacement policies. Discuss the importance of replacement models.

(4 × 4 = 16 marks)

Part B

Answer **either** *Part A* **or** *Part B of* **all** *questions.*
Each question carries 16 *marks.*

II. (A) (a) Derive the necessary and sufficient condition for the optimal solution of $z = f\left(x_1, x_2, ..., x_n\right)$.

Subject to $g\left(x_1, x_2, ..., x_n\right) \le c$ and $x_1, x_2, ..., x_n \ge 0$ where c is a constant.

(b) Solve the NLPP,

Maximize $z = 2x_1 - x_1^2 + x_2$ subject to the constraints :

$$2x_1 + 3x_2 \le 6,\ 2x_1 + x_2 \le 4,\ x_1, x_2 \ge 0.$$

Or

Turn over

(B) (a) Derive the Kuhn-Tucker condition for solving a QPP.

(b) Use Wolf's method to solve the QPP :

Maximize $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$ subject to the constraints :
$x_1 + 2x_2 \leq 2, x_1, x_2 \geq 0.$

III. (A) (a) Find u_j which minimize :

$$z = \sum_{j=1}^{n} f\left(u_j\right) \text{ subject to the constraints } \sum_{j=1}^{n} a_j\, u_j \geq b, a_j, b_j, u_j \in (0, \infty).$$

(b) A student has to take an examination in three courses X, Y and Z. He has three days for studying. He feels it would be better to devote a whole day to study one single course. So he may study a course for one day, two days or three days or not at all. His estimates of grades he may get according to days of study he puts in are as follows :

Study Days	Courses		
	X	Y	Z
0	1	2	1
1	2	2	2
2	2	4	4
3	4	5	4

How many days should he allocate to each course so that he gets the best results ?

Or

(B) (a) Explain GPP.

(b) Minimize $f(x) = \dfrac{c_1}{x_1x_2x_3} + c_2x_2x_3$ subject to $g(x) = c_3x_1x_3 + c_4x_1x_2, c_i > 0, x_i > 0.$

IV. (A) (a) Explain production lot size model.

(b) The demand for a certain item is 16 units per period .Unsatisfied demand causes a shortage of cost Rs. 0.75 per unit per short period. The cost of initiating purchasing action is Rs. 15 per purchase and the holding cost is 15% of average inventory valuation per period. Item cost is Rs. 8 per unit (Assume that the shortages are being back ordered at the above mentioned cost). Find the minimum cost purchase quantity.

Or

(B) (a) Distinguish between deterministic and probability model.

(b) Describe a single period probabilistic inventory model and derive an expression for the optimal order quantity Q.

V. (A) (a) State and prove mortality theorem.

(b) A firm is considering replacement of a machine whose cost price is Rs. 12,200 and the scrap value Rs. 200. The running (maintenance and operating) cost in rupees are found from experience as follows :

Year	:	1	2	3	4	5	6	7	8
Running Cost	:	200	500	800	1200	1800	2500	3200	4000

When should the machine be replaced ?

Or

(B) (a) How do you generate random numbers using Monte-Carlo simulation method ?

(b) Write a brief note on various simulation techniques.

(4 × 16 = 64 marks)

C 2087 (Pages : 3) Name.....................................

Reg. No.................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION MARCH 2021

(CBCSS)

Statistics

MST 4E 18—DATA MINING TECHNIQUES

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries 2 *weightage.*

1. What is meant by agglomerative nesting ?
2. Mention the use of Information measures in decision trees.
3. Why k-nearest neighborhood method is called lazy learner ?
4. What is meant by dimension reduction.?
5. Mention the role of activation functions in ANN.
6. Define : Confidence of a rule.
7. Distinguish between association analysis and correlation analysis.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Explain any two hierarchical clustering methods.
9. Mention how the choice of initial centroids is determined in k-means clustering.

Turn over

10. Explain Naive Bayes classification.

11. Explain the application of sigmoid and softmax activation functions.

12. Explain any two operations one can perform in RDBMS with examples.

13. Write a note on apriori algorithm in association analysis.

14. Write a descriptive note on analytical data processing.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Perform single linkage clustering using the following data, produce the dendrogram and also compute the agglomerative coefficient :

	{SJ}	{SO}	{RM}	{TP}	{KK}
{SJ}	0				
{SO}	37.93	0			
{RM}	12.72	42.39	0		
{TP}	27.98	33.11	37.64	0	
{KK}	22.07	44.09	13.45	47.07	0

16. Given the training data set :

S.No.	Region	Literary Level	Female population	Turnout percentage	Winning Party
1	Urban	High	Low	High	A
2	Semi Urban	High	High	High	B
3	Rural	Low	High	Low	B
4	Urban	High	Low	Low	B
5	Rural	Low	High	Medium	A
6	Urban	Medium	High	Low	B
7	Rural	Low	High	Medium	A

S.No.	Region	Literary Level	Female population	Turnout percentage	Winning Party
8	Rural	Medium	High	Low	B
9	Urban	High	Low	High	A
10	Semi Urban	Low	High	High	B
11	Semi Urban	Medium	High	Low	B
12	Urban	Medium	Low	Low	B
13	Rural	Low	Low	High	A
14	Semi Urban	Medium	High	Low	A
15	Rural	High	Low	Medium	B
16	Urban	Low	High	Low	B

Predict the outcomes corresponding to (a) (Rural, Low, Low, Low) and (b) (Semi Urban, High, Low, Medium) using Naive Bayesian classification.

17. Explain the operational details of an artificial neural network.

18. Write a descriptive note on the application of data mining tools in electronic commerce.

(2 × 5 = 10 weightage)

C 2086 (Pages : 2) Name.......................................

Reg. No...................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION MARCH 2021

(CBCSS)

Statistics

MST 4E 08—RELIABILITY MODELLING

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each answer carries 2 weightage.

1. Obtain the structure function of a k-out-of-n system.
2. Define the reliability of a system of n components.
3. What is the concept of no ageing in Reliability ?
4. If F is NBUE, then show that $\int_0^t \overline{F}(x)\,dx \geq \mu F(t)$, where $\mu = \int_0^\infty \overline{F}(x)\,dx$, the mean of the distribution.
5. Define type I censoring in Reliability estimation.
6. How do you compute the Reliability of a stress-strength model ?
7. Distinguish between Availability and Reliability.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Each answer carries 3 weightage.

8. Prove that no system can perform better than a parallel system and worse than a series system.

Turn over

9. Write the minimal paths and cuts of a 5 component Sterio-Hi-fi-system, functioning with a tuner, Recorder, amplifier and two speakers. Draw the corresponding bridge figure of the system.

10. What is the relevance of Gamma distribution in Reliability analysis ?

11. Prove or disprove DMRL $\Rightarrow$ NBU.

12. Obtain the MLE and UMVUE of the parameter when the life distribution follows the one parameter exponential distribution for a failure censored data with replacement.

13. Discuss the Bivariate Exponential distribution as a shock model receiving fatal shocks from two independent external sources.

14. What is the maintainability function when the repair time follows the exponential distribution ?

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*

Each question carries 5 *weightage.*

15. Explain the concept of explicit bounds of system reliability in terms of the component reliabilities in a Coherent system.

16. State and prove the DFR and DFRA closure property in mixture of distributions.

17. Show that the only BVED satisfying the bivariate lack of memory property is that discussed in question number 13.

18. What is the problem of testing exponentiality against positive ageing ? Discuss the Deshpande's test for exponentiality.

(2 × 5 = 10 weightage)

C 2085 (Pages : 2) Name.......................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION MARCH 2021

(CBCSS)

Statistics

MST 4E 07—STATISTICAL DECISION THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries a weightage of 2.

1. Distinguish between randomized decision rule and non-randomized decision rule.
2. Define loss function and risk function.
3. Define Jeffreys prior. How is the Jeffreys prior useful ?
4. State Baye's theorem.
5. Define a decision rule and R-better decision rule.
6. Distinguish between pure strategy and mixed strategy in a two-person zero sum game.
7. Show that under squared-error loss, the Bayes rule is the mean of the posterior distribution.

($4 \times 2 = 8$ weightage)

Part B

Answer any **four** *questions.*
Each question carries a weightage of 3.

8. Express hypothesis tests and problem of estimation as finite action decision problem.
9. Explain the terms : (i) Prior ; (ii) Posterior ; (iii) Risk ; and (iv) Baye's Risk.

Turn over

10. Prove that if a minimax estimator of a parametric function is unique, then it must be admissible.

11. Show that for binomial family, the conjugate prior is beta distribution.

12. Discuss the basic elements of game theory.

13. Discuss the steps involved in the construction of utility function.

14. Show that Baye's rule with constant risk is minimax.

($4 \times 3 = 12$ weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

15. Explain the equalizer rule of determining minimax decision rule. Using this method obtain the minimax estimator for θ in binomial (n, θ) under squared error loss.

16. Define conjugate priors. Find Baye's estimator and Bayes risk if $X \sim N(\mu, 1)$ under absolute error loss function using natural conjugate prior for μ. Examine whether this estimator is minimax.

17. Discuss any one general technique of solving an $m \times n$ game.

18. Describe the various approaches in the subjective determination of the prior density.

($2 \times 5 = 10$ weightage)

C 2084 (Pages : 3) Name.......................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION MARCH 2021

(CBCSS)

Statistics

MST 4E 01—OPERATIONS RESEARCH

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Weightage 2 for each question.

1. Explain the essential characteristics of linear programming model.
2. Define : (i) basic solution ; (ii) feasible solution ; (iii) basic feasible solution ; and (iv) degenerate solution in the context of LPP.
3. Prove that dual of the dual is primal.
4. What is sequencing problem ? What are the assumptions underlying a sequencing problem ?
5. Define : (i) pure strategy ; (ii) mixed strategy ; (iii) saddle point ; and (iv) pay-off matrix with reference to game theory.
6. Write short notes on post optimal sensitivity analysis.
7. What are slack and surplus variables ? Write down the standard form of LPP.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Weightage 3 *for each question.*

8. Define transportation problem and give its mathematical model. Establish the necessary and sufficient condition for the existence of feasible solution to a transportation problem.

9. What are artificial variables ? Describe the two phase method of solving LPP.

10. Determine the optimum of sequence of jobs that minimizes the total elapsed time based on the following information processing time on machines is given in hours and passing is not allowed

Job	J_1	J_2	J_3	J_4	J_5	J_6	J_7
Machine A	3	8	7	4	9	8	7
Machine B	4	3	2	5	1	4	3
Machine C	6	7	5	11	5	6	12

11. Distinguish between pure and mixed integer programming problems. Explain the procedure of constructing Gomory's constraint in all integer programming problem.

12. If an LPP has a feasible solution, show that it also has a basic feasible solution.

13. Explain the theory of dominance in the solution of rectangular games. Illustrate with an example.

14. State the fundamental theorem of duality and explain dual simplex method of solving linear programming problem.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Weightage 5 *for each question.*

15. (i) Using graphical method, find the maximum value of $Z = 50x_1 + 60x_2$ subject to the constraints $2x_1 + 3x_2 \leq 1500, 3x_1 + 2x_2 \leq 1500, 0 \leq x_1 \leq 400, 0 \leq x_2 \leq 400$.

(ii) Write short notes on the role of pivot element in a simplex method. How do you recognize optimality ?

16. (i) Explain the Vogel's Approximation method to obtain an initial basic feasible solution to a transportation problem.

(ii) Four operators A, B, C and D are available to a manager who has to gel lour jobs I, II, III and IV done by assigning one job to each operator. The times needed by different operators for different jobs in the matrix given below :

		Operator			
		A	B	C	D
Job	I	15	13	14	17
	II	11	12	15	13
	III	18	12	10	11
	IV	15	17	14	16

How should the manager assign jobs so that the total time needed for all the jobs is minimum ?

17. (i) Explain briefly the Maximin-Minimax principle used in game theory.

(ii) Show how a two person zero sum can be reduced to a linear programming problem.

(iii) Describe graphical method of solving $2 \times n$ games.

18. Write short notes on the following :

(i) Economic interpretation of duality.

(ii) Degeneracy in LPP.

(iii) Zero one programming problem.

(iv) Applications of integer programming problem.

($2 \times 5 = 10$ weightage)

C 2083 (Pages : 2) Name..................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION MARCH 2021

(CBCSS)

Statistics

MST 4C 14—MULTIVARIATE ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries a weightage 2.

1. If $(X_1 \ X_2 \ X_3)'$ is a trivariate Normal vector find the distribution of $\frac{(X_1 + X_2 + X_3)}{3}$.
2. State the Cochran's theorem for the independence of quadratic forms.
3. Distinguish between multiple correlation and canonical correlation.
4. Derive the characteristic function of a non-singular multivariate Normal distribution.
5. Define generalised variance. What is its relevance in multivariate analysis ?
6. Write down Hotelling's T^2 statistic as a likelihood ratio criterion.
7. What do you mean by factor loadings in Factor Analysis ?

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Each question carries a weightage 3.

8. With usual notations prove that $r_{12.3} = \frac{r_{12} - r_{12} r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}}$.

9. Let $X \sim N_p(\mu, \Sigma)$ and it is partitioned as $X = \begin{pmatrix} X^{(1)} \\ X^{(2)} \end{pmatrix}$. Define $Y^{(1)} = X^{(1)} + BX^{(2)}$ and $Y^{(2)} = X^{(2)}$.

   Determine B such that and $Y^{(1)}$ and $Y^{(2)}$ are independent. Also write the conditional distributions.

10. Obtain the MLE of μ and Σ when sampling from Multivariate Normal population with parameters μ and Σ.

11. If $A \sim W_p(n, \Sigma)$, a Wishart distribution, then show that $L' AL \sim \sigma_l^2 \chi^2(n)$,

    where $L' \Sigma L = \sigma_l^2$, L is a vector of real constants order $p \times l$.

12. Establish the invariance property of Mahalanobis D^2.

13. Explain the test for independence of sets of variables of a multivariate normal population.

14. Describe an iterative procedure for the construction of the principal components.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage 5.

15. Let $X \sim N_p(0, \Sigma)$, then write the necessary and sufficient condition for the independence of the quadratic forms $X' AX$ and $X' BX$ where A and B are real symmetric matrices.

16. Derive the null distribution of the partial correlation co-efficient.

17. Derive the distribution of Hotelling's T^2 statistics.

18. Explain the problem of classification into one of the several multivariate normal populations with (a) Common Dispersion matrix ; and (b) unequal dispersion matrices.

(2 × 5 = 10 weightage)