D 92961-A **(Pages : 4)** Name.......................................

Reg. No...................................

THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2020

Mathematics

MEC 3C 03—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MEC 3C 03—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. When $Y''(t) - 10, Y(t)$ will be :

   (A) 1.
   (B) 0.
   (C) $5t^2$.
   (D) $5t^2 + tc_1 + C$.

2. What is the degree of $\left(\frac{dy}{dx^2}\right)^6$.

   (A) 2.
   (B) 3.
   (C) 4.
   (D) 6.

3. Given $Q_d = 6 - 2P$ $Q_s = 4 + 4P$ equlibrium $\overline{p}$ will be :

   (A) 6.
   (B) 4.
   (C) 10.
   (D) $1/3$

4. Law of returns to scale is a theory pertaining to :

   (A) Market period.
   (B) Short period.
   (C) Long period.
   (D) None of the above.

5. If $\frac{dy}{dt} = 15$, the value of $Y_{(t)}$ is :

   (A) 15.
   (B) 0.
   (C) $15t$ + A.
   (D) None of the above.

6. Which of the following is used for constrained optimization ?

   (A) Hessian.
   (B) Barclered Hessian.
   (C) Discriminant.
   (D) Jacobian.

7. The value of Y when $\frac{dy}{dt} = Y^2 t$ is :

(A) $\frac{2}{t+c}$.

(B) $\frac{-2}{t+c}$.

(C) $\frac{1}{t^2+c}$.

(D) $\frac{-2}{t^2+c}$.

8. Production function shows technological relationship between input and ————.

(A) Output

(B) Factors of production.

(C) Both of the above.

(D) None of the above.

9. When demand for a good is give by Q = 40 – P, the maximum amount that would be demanded at nil price is :

(A) 1.

(B) 0.

(C) 40.

(D) 400.

10. Technical relationship between input and output is called :

(A) Elasticity.

(B) Production function.

(C) Input function.

(D) None of the above.

11. For the production function $Q = AL^{0-4} L^{05}$, which of the following is true ?

(A) Increasing returns to scale.

(B) Diminishing returns to scale

(C) Constant returns to scale.

(D) Variable returns to scale.

12. What is the order of $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^6 = 10 - Y$.

(A) First order.

(B) Second order.

(C) Third Order.

(D) Fourth order

13. What is the degree of $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^6 = 10 - Y$.

(A) 3.

(B) 4.

(C) 2.

(D) 6.

14. The producer will be in equilibrium when :

(A) MRTS $= \frac{p_x}{p_y}$.

(B) MRTS $> \frac{p_x}{p_y}$.

(C) MRTS $< \frac{p_x}{p_y}$.

(D) None of the above.

15. The longrun theory of output behavior is known as :

(A) Diminishing returns.

(B) Law of variable proportions.

(C) Diminishing Marginal products.

(D) Law of returns to scale.

16. Select the odd one among land, capital, organization, profit :

(A) Land.

(B) Capital.

(C) Organisation.

(D) Profit.

17. In the production function $Q = r\left[\delta C^{-\propto} + (1-\delta) N^{-\propto}\right]^{\frac{-V}{\propto}}$, efficient is measured with :

(A) δ.

(B) r.

(C) N.

(D) C.

18. The value of Rs. 100 at 10 % interest for two years :

(A) 110.

(B) 111.

(C) 121.

(D) 130.

19. Functional relationship between input and output is called :

(A) Isoquants..

(B) Isocost.

(C) Input function.

(D) Production function.

20. What is the degree of $\frac{d^3Y}{dx^3} + x^2Y\left(\frac{d^2Y}{dx^2}\right) - 4Y^4 = 1$.

(A) First.

(B) Second.

(C) Third.

(D) Fourth.

D 92961

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Name.....................................

Reg. No.................................

THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2020

Mathematics

MEC 3C 03—MATHEMATICAL ECONOMICS

Time : Two Hours | Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*

Each question carries 3 marks.

All questions can be attended.

Overall Ceiling 24.

1. Distinguish between the order and degree of a difference equation with example.
2. What do you mean by CES production function ?
3. Discuss the concept of Expansion Path.
4. What is Economic Region of production ?
5. Discuss the concept of Returns to Scale.
6. Limitations of Cobb-Douglas Production Function.
7. How will you demonstrate Producer's Equilibrium ?
8. What is elasticity of substitution ?
9. Why do you think that investment appraisal is necessary ? What is Internal Rate of Return ?
10. What is the difference between risk and uncertainty in Economics ?
11. What do you mean by sensitivity analysis ?
12. What do you mean by separation of Variables ?

(8 × 3 = 24 marks)

Turn over

Section B

Answer at least **five** *questions.*

Each question carries 5 *marks.*

All questions can be attended.

Overall Ceiling 25.

13. Solve the first order homogenous linear difference equation $y_{x+1} = 4y$.

14. Explain the Probability Distribution approach in evaluation risk.

15. Derive the formula for the value P_t of an initial amount of money Po deposited at i interest for t years when compounded annually.

16. Explain the Lagged Income Determination model using difference equations.

17. Briefly describe the economic application of differential equations in Economics.

18. What is the difference between Risk adjusted discount rate (RAD) approach and Certainty-Equivalent approach ?

19. Explain homogeneous production function.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*

Each question carries 11 *marks.*

20. What is production function? Explain the Law of Variable Proportions.

21. Optimise the generalized Cobb-Douglas production function $Q = K^{0.3} L^{0.5}$ subject to $6K + 2L = 384$.

(1 × 11 = 11 marks)

D 92960 (Pages : 3) Name..

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THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MTS 3C 03—MATHEMATICS – 3

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. If $\bar{r}(t) = 2\cos t\,\hat{i} + 6\sin t\,\hat{j}$, find $\frac{d\bar{r}}{dt}$ at $t = \frac{\pi}{2}$.

2. Find the curvature of a circle whose radius is 2.

3. If $z = e^x \sin(xy)$, find $\frac{\partial^2 z}{\partial y^2}$.

4. Find the gradient of $f(x, y, z) = xy^2 + 3x^2 - z^3$ at (1, 1, 1).

5. Show that div $\vec{r} = 3$.

6. Evaluate $\int_2^4 \int_1^3 (40 - 2xy)\, dx\, dy$.

7. Use double integrals to find the area of the plane region enclose by the curves $y \sin x$ and $y = \cos x$ for $0 \le x \le \pi/4$.

8. Find the Jacobian of $u = \frac{y}{x^2}, v = xy$.

9. Sketch the graph of the region $|z - 2i| = 2$.

10. Write the real and imaginary part of $f(z) = \sin z$.

Turn over

11. Evaluate $\oint_C \frac{z^2}{z-1}\, dz$, where C is $|z| = 2$.

12. Evaluate $\int_C z\, dz$ where C is given by $x = t^2, y = t$ from $0 \le t \le 1$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Find the directional derivative of $F(x,y,z) = xy^2 - 4x^2y + z^2$ at $(1,-1,2)$ in the direction of $6\hat{i} + 2\hat{j} + 3\hat{k}$.

14. Find an equation of the tangent plane to the graph of $x^2 - 4y^2 + z^2 = 16$ at (2, 1, 4).

15. Use Green's theorem to evaluate $\oint_C (x^2 - y^2)\, dx + (2y - x)\, dy$, where C consists of the boundary of the region in the first quadrant that is bounded by $y = x^2$ and $y = x^3$.

16. Change the order of integration and hence evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$.

17. Use divergence theorem to evaluate $\iint_S (\vec{F} \cdot \hat{n})\, dS$ where $\vec{F} = xy\,\hat{i} + y^2 z\hat{j} + z^3\hat{k}$ and S is the unit cube defined by $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$.

18. Evaluate $\oint_C \left(z + \frac{1}{z}\right) dz$, where C is the unit circle $|z| = 1$.

19. Evaluate $\oint_C \frac{z^4 - 3z^2 + 6}{(z+i)^3}\, dz$, where C is $|z| = 2$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Use Stoke's theorem to evaluate $\oint_C z\,dx + x\,dy + y\,dz$, where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$ counter clockwise as viewed from above.

21. Find the volume of the solid in the first octant bounded by the graphs of $z = 1 - y^2$, $y = 2x$ and $x = 3$.

(1 × 11 = 11 marks)

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Reg. No..................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(Multiple Choice Questions for SDE Candidates)

1. $\int \frac{\ln x}{x}\, dx =$ ————.

(A) $\frac{1}{x^2} + c.$

(B) $\frac{(\ln x)^2}{2} + c.$

(C) $\frac{\ln x}{2} + c.$

(D) $\frac{1}{x} - \ln x + c.$

2. $\frac{d}{dn}\left(a^x\right) =$ ————.

(A) $xa^{x-1}.$

(B) $a^x.$

(C) $a^x \log a.$

(D) $\frac{a^x}{\log a}.$

3. $\int_1^{\sqrt{2}} x 2^{x^2}\, dx =$ ————.

(A) 2.

(B) $\frac{1}{\ln 2}.$

(C) $\frac{\ln 2}{2}.$

(D) $\frac{2}{\ln 2}.$

4. Range of $\cos hx$ is ————.

(A) $[-1, 1].$

(B) $(-\infty, \infty).$

(C) $(-\infty, 1].$

(D) $(-\infty, 1).$

5. $\coth h^{-1} x =$ ________.

(A) $\frac{1}{\tanh x}$.

(B) $\tanh^{-1}\left(\frac{1}{x}\right)$.

(C) $\coth^{-1}\left(\frac{1}{x}\right)$.

(D) $\frac{1}{\tan^{-1}\left(\frac{1}{x}\right)}$.

6. $\lim_{x \to 1} \frac{1-x}{\ln x} =$ ________.

(A) -1.

(B) $+1$.

(C) 0.

(D) 2.

7. $\lim_{x \to 0} \frac{a^x - b^x}{x} =$ ________.

(A) $\ln\left(\frac{a}{b}\right)$.

(B) $\ln\left(\frac{b}{a}\right)$.

(C) $\ln(ab)$.

(D) ∞.

8. $\lim_{n \to \infty} \left(\frac{n-1}{n}\right) =$

(A) 0.

(B) -1.

(C) 1.

(D) ∞.

9. $\lim_{n \to \infty} \frac{\ln n}{n} =$ ________.

(A) 0.

(B) 1.

(C) ∞.

(D) None of these.

10. If $|r| < 1$, $\lim\limits_{n \to \infty} r^n =$ ————

(A) 0.

(B) 1.

(C) ∞.

(D) None of these

11. Let $\{a_n\}$ be a sequence of positive terms such that $a_n \geq a_{n+1}$ and $\lim\limits_{n \to \infty} a_n = 0$, then :

(A) $\sum a_n$ converges.

(B) $\sum (-1)^n a_n$ converges.

(C) $\sum a_n$ converges but $\sum (-1)^n a_n$ diverges.

(D) None of these.

12. If the series $\sum a_n (x-c)^n$ converges for $x = k$. Then it converges in :

(A) $|r - c| < k$.

(B) $|r - c| \leq k$.

(C) $|r - c| > k$.

(D) $|r - c| \geq k$.

13. The radius of convergence of the series $\sum\limits_{n=0}^{\infty} x^n$ is ————.

(A) 1.

(B) 2.

(C) $\sqrt{2}$.

(D) ∞.

14. Co-efficient of $(x-1)^2$ in the Taylor series expansion of $f(x) = \dfrac{1}{x-1}$ at $x = 2$ is :

(A) -1.

(B) ½.

(C) $-$ ½.

(D) $+1$.

15. Which of the following point lie on the curve $r = -\sin\left(\frac{\theta}{3}\right)$.

(A) $\left(\frac{1}{2}, \frac{3\pi}{2}\right)$.

(B) $\left(\frac{1}{2}, \frac{\pi}{2}\right)$.

(C) $\left(1, \frac{3\pi}{2}\right)$.

(D) None

16. Polar equation of a circle passing through origin, radius 2 and centre on +ve x-axis is :

(A) $r = 4\sin\theta$.

(B) $r = 4\cos\theta$.

(C) $r = 2\cos\theta$.

(D) $r = -2\cos\theta$.

17. $\int_0^{\pi/6} \tan 2x \, dx =$ ________.

(A) $\frac{1}{\ln 2}$.

(B) $\frac{1}{2}\ln 2$.

(C) $2\ln 2$.

(D) $\frac{2}{\ln 2}$.

18. $\log_a\left(a^x\right) =$ ________.

(A) a^x.

(B) a.

(C) x^a.

(D) x.

19. The series $\sum n^m x^n$ is converged if ———.

(A) $x > 1$ and $x = 1$ when $m < -1$.

(B) $x > 1$ and $x = 1$ when $m > -1$.

(C) $x < 1$ and $x = 1$ when $m < -1$.

(D) $x < 1$ and $x = 1$ when $m > -1$.

20. $x = a\sec t, y = b\tan t$ is the parametric representation of :

(A) Parabola.

(B) Ellipse.

(C) Hyperbola.

(D) Circle.

D 92959 (Pages : 3) Name.....................................

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THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find the derivative of $y = e^{-x\cos x}$.

2. Find the derivative of $y = \log\left(e^{2x} + e^{-2x}\right)$.

3. Find $\lim\limits_{x \to 1^{+}} \dfrac{\sin \pi x}{\sqrt{x-1}}$.

4. Show that $\sinh^{-1} x = \log\left(x + \sqrt{x^2+1}\right)$.

5. Find $\lim\limits_{n \to \infty} e^{\sin\left(\frac{1}{n}\right)}$.

6. Express $3.\overline{214}$ as a rational number.

7. Determine whether the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{3+2^n}$ converges or diverges.

8. Show that the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{\sqrt{n}+1}$ is divergent.

9. Find a power series representation of $\dfrac{1}{(1-x)^2}$ on (–1, 1) by differentiating a power series representation of $\dfrac{1}{1-x}$.

Turn over

10. Find the Maclaurin series of $\sin x$ and determine its interval of convergence.

11. Find a rectangular equation whose graph contains the curve C with the given parametric equation $x = 2t + 1, y = t - 3$.

12. Find the equation of the tangent line to the curve $x = \sec t, y = \tan t, \frac{-\pi}{2} < t < \frac{\pi}{2}$ at $t = \pi/4$.

13. Find the parametric equation for the line passing through the point (–2, 1, 3) and parallel to the vector $< 1,2,-2)$.

14. The point $\left(3, \frac{\pi}{3}, \frac{\pi}{4}\right)$ is expressed in spherical co-ordinates. Find its rectangular co-ordinates.

15. Find the antiderivative of $r^1(t) = \cos t i + e^{-t} j + \sqrt{t}\ k$ satisfying the initial condition $r(0) = i + 2j + 3k$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Use logarithmic differentiation to find the derivative of $y = \left(\sqrt{\cos x}\right)^x$.

17. Evaluate $\lim\limits_{x \to 0} \frac{x^3}{x - \tan x}$.

18. Evaluate $\int\limits_{-\infty}^{0} xe^x dx$.

19. Show that the series $\sum\limits_{n=1}^{\infty} \left[\frac{2}{n(n+1)} - \frac{4}{3^n}\right]$ is convergent and find its sum.

20. Find the radius of convergence and interval of convergence of the series $\sum\limits_{n=1}^{\infty} \frac{(x-2)^n}{n^2 \cdot z^n}$.

21. Find the Taylor series for $f(x) = \log x$ at $x = 1$ and determine its interval of convergence.

22. Identify and sketch the surface $4x - 3y^2 - 12z^2 = 0$.

23. Let C be the helix $r(t) = 2\cos t\ i + 2\sin t\ j + t\ k, t \geq 0$. Find T (t) and N (t).

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Find $\int \frac{e^x}{e^{2x}+1}\,dx$.

(b) Find $\int \cosh(2x+3)\,dx$.

(c) Evaluate $\lim_{x \to 0} x^x$.

25. (a) Evaluate $\int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$.

(b) Determine whether the series $\sum_{n=1}^{\infty} \frac{2n^2+n}{\sqrt{4n^7+3}}$ converges or diverges.

26. Find the area of the region that lies outside the circle $r = 3$ and inside the cardioid $r = 2 + 2\cos\theta$.

27. A shell is fired from a gun located on a hill 100 m above a level terrain. The muzzle speed of the gun is 500 m/sec and its angle of elevation is 30.

(a) Find the range of the shell.

(b) What is the maximum height attained by the shell ?

(c) What is the speed of the shell at impact ?

(2 × 10 = 20 marks)

D 91752-A (Pages : 4) Name.......................................

Reg. No.......................................

THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2020

Mathematics

ME 3C 03—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

ME 3C 03—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. Curve that is also known as equal product curve is :

   (A) Indifference curve.
   (B) Isoquants.
   (C) Demand Curve.
   (D) None of the above.

2. If $\frac{dy}{dt} = 15$, the value of $Y_{(t)}$ is :

   (A) 15.
   (B) 0.
   (C) $15t + A$.
   (D) None of the above.

3. The third stage in the law of variable proportion is called :

   (A) Increasing returns.
   (B) Diminishing returns.
   (C) Negative returns.
   (D) Proportional return.

4. In the Cobb-Douglas production function $AK^{\alpha}L^{\beta}$, A denotes :

   (A) Inputs.
   (B) Output.
   (C) Efficiency parameter.
   (D) None of the above.

5. Production function shows technological relationship between input and ————.

   (A) Output.
   (B) Factors of production.
   (C) Both of the above.
   (D) None of the above.

6. Marginal product equals :

   (A) $\frac{TP}{p}$.
   (B) $\frac{AP}{p}$.
   (C) Slope of TP curve.
   (D) Slope of AP curve.

7. Slope of Iso-quant is called :

   (A) MRS.
   (B) MRTS.
   (C) MP.
   (D) AP.

8. Euler's theorem is valid only for ———— function.

   (A) Non-linear.
   (B) Linear.
   (C) Quadratic.
   (D) Exponential.

9. Law of diminishing returns is also known as :

   (A) Variable proportion.
   (B) Returns to scale.
   (C) Isoquant.
   (D) Price line.

10. Higher isoquants represents higher :

   (A) Profit.
   (B) Output.
   (C) Cost.
   (D) None of the above.

11. What is the order of differential equation $\frac{dy}{dt} = 10x + 5$?

   (A) First.
   (B) Second.
   (C) Third.
   (D) Fourth.

12. Given the demand curve P = 10 – 0.2 Q, total revenue curve :

   (A) 1 – 0.2 Q.
   (B) $10\ Q^2 - 0.2\ Q$.
   (C) 10 Q – 0.2 Q.
   (D) $10\ Q - 0.2\ Q^2$.

13. What is the degree of $\left(\frac{dy}{dt}\right)^4 - 6t^5$?

   (A) First degree.
   (B) Second degree.
   (C) Third degree.
   (D) Fourth degree.

14. Demand function $Q = f(P)$ if point elasticity $\epsilon = -1$ for all P > 0 will be :

   (A) CP.
   (B) $\frac{c}{p}$.
   (C) P.
   (D) None of the above.

15. What is the order of $\left(\frac{d^2y}{dt^2}\right)^7 + \left(\frac{d^2y}{dt^3}\right)^5 = 100y$?

(A) 2.
(B) 3.
(C) 5.
(D) 7.

16. The degree of $\frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^3 + 25 = 0$ is :

(A) First.
(B) Second.
(C) Third.
(D) Fourth.

17. Law of variable proportion explains for :

(A) Short run.
(B) Long run.
(C) Medium.
(D) None of the above.

18. In the Cobb-Douglas production function $AK^{\alpha}L^{\beta}$, $\propto$ denotes :

(A) Labour share.
(B) Capital share.
(C) Output.
(D) Input.

19. If both factors X_1 and X_2 are perfect substitutes, then the value of elasticity of substitution is :

(A) 1.
(B) 0.
(C) $0 < \sigma < 1$.
(D) Infinity.

20. Which of the following is not a feature of Cobb-Douglas production factor ?

(A) Linearity.
(B) Homogenecity.
(C) Constant returns.
(D) Increasing returns.

D 91752 (Pages : 4) Name.....................................

Reg. No.................................

THIRD SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION NOVEMBER 2020

Mathematics

ME 3C 03—MATHEMATICAL ECONOMICS

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. What is the order of the differential equation $\frac{d^2y}{dx^2} + \frac{d^4y}{dx^4} - \left(\frac{dy}{dx}\right)^2$?

   (a) 4. (b) 3. (c) 2. (d) 1.

2. Write a differential with first order and 2nd degree.

3. Give an example of exact differential equation.

4. In the Cobb-Douglas production function $AK^{\alpha}L^{\beta}$, A denotes :

   (a) Inputs. (b) Output.

   (c) Efficiency parameter. (d) None of the above.

5. When the total product is maximum, marginal product will be :

   (a) Minimum. (b) Zero.

   (c) Maximum. (d) Negative.

6. Slope of ISo-quant is called :

   (a) MRS. (b) MP.

   (c) AP. (d) MRTS.

7. Technical relationship between input and output is called :

   (a) Elasticity. (b) Production function.

   (c) Input function. (d) None of the above.

8. Demand function Q = f(P), the variable P denotes :

   (a) Product. (b) Production.

   (c) Price. (d) Profit.

Turn over

9. In CES production function, the elasticity of substitution is :

(a) Unity. (b) Zero.

(c) Negative. (d) Constant.

10. A line that connects various equilibrium points of producer is :

(a) Isocost line. (b) Isoquants.

(c) Expansion path. (d) Price line.

11. If both factors X_1 and X_2 are perfect substitutes, then the value of elasticity of substitution is :

(a) 1. (b) 0.

(c) $0 < \sigma < 1$. (d) Infinity.

12. Consumer will be at equilibrium when he maximizes :

(a) Profit. (b) Output.

(c) Satisfaction. (d) Income.

(12 × 1 = 12 marks)

Part B

Answer any **six** *questions in two* **or** *three sentences.*
Each question carries 3 marks.

13. Find the general solution of $\left(6xy + 9y^2\right)dy + \left(3y^2 + 8x\right)dx = 0$.

14. Solve the difference equation $y_t = \frac{1}{8}y_{t-1}$.

15. Solve $8y_{t-2} - 2y_{t-3} = 120$ and $y_0 = 28$.

16. Write a short note on isoquants.

17. Explain briefly 'Law of variable proportions'.

18. What are the assumptions made in the Euler's Theorem ?

19. What are the main drawbacks of Cobb - Douglas production function.

20. Write a short note on Payback method of investment appraisal.

21. Write any three similarities between Net Present Value method and Internal Rate of Return method.

(6 × 3 = 18 marks)

Part C

Answer any **six** *questions from the following.*
Each question carries 5 *marks.*

22. Solve by finding the integrating factor : $4xdy + \left(16y - x^2\right)dx = 0.$

23. Derive the formula for the total value of an initial sum of money P (0) set out for t years at interest rate i, when i is compounded continuously.

24. Find the firms expansion path if the production function is $q = 12\log x_1 + 30\log x_2$ and the input price of unit values of x_1 and x_2 respectively are P_1 and P_2 and $P_1 = 2, P_2 = 5$.

25. Given the equation for a production isoquant $16K^{1/4}L^{3/4} = 2144$. Find the marginal rate of technical substitution.

26. Given $Q = 4\sqrt{KL},$ find (a) MP_K and MP_L and (b) determine the effect on Q of a 1-unit change in K and L, when K = 50 and L = 600.

27. Use the properties of homogeneity to show that a strict Cobb-Douglas production function $q = AK^{\alpha}L^{\beta}$, where $\alpha + \beta = 1$, exhibits constant returns to scale.

28. A project of Rs. 20 lakhs yielded annually a profit of Rs. 4 lakhs after depreciation at 10% and is subject to income tax at 40%. Calculate payback period of this project.

29. Write the main advantages and limitations of Net Present Value (NPV) method.

30. The expected NPV of a project has been worked out as Rs. 23,206 and its standard deviation is Rs. 9,343. If the total distribution is approximately normal and assumed continuous, then determine the probability of NPV under the following ranges : (i) Zero or less ; (ii) Greater than zero.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions from the following.*
Each question carries 10 *marks.*

31. For the data given below, determine (a) the market price Pt in any time period, (b) the equilibrium price Pe, and (c) the stability of the time path : $Q_{dt} = 180 - 0.75\,P_t, Q_{st} = -30 + 0.3P_{t-1}, P_0 = 220$.

32. Given a budget constraint of Rs.108 when $P_K = 3$ and $P_L = 4$. Optimize the generalized Cobb-Douglas production function $q = AK^{0.4}L^{0.5}$.

33. Determine the Internal Rate of Return (IRR) to the nearest percent for a project requiring an initial outlay of Rs. 10,000 resulting in a cash flow of Rs. 2,000 at the end of years one through five and Rs. 5,000 at the end of year six.

34. The probability distributions of NPV's of two investment projects A and B are as follows :

Project A		Project B	
NPV (Rs.)	Probability	NPV (Rs.)	Probability
5,000	0.20	0	0.1
7,500	0.70	7500	0.7
10,000	0.10	15000	0.2

(i) Compute the expected NPV of these projects ; (ii) Compute the risk attached to each points ; (iii) Which project is more risky ? Why ?

(2 × 10 = 20 marks)

D 91751 (Pages : 4) Name.......................................

Reg. No...................................

THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2020

Mathematics

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. What do you mean by a homogeneous equation ?
2. Consider a system of linear equations in n unknowns with augmented matrix $M = [A, B]$. Then, the solution is unique if and only if rank (A).
3. What is the order of the differential equation $y\left(\frac{dy}{dx}\right)^2 + 8x = 0$.
4. State Cayley-Hamilton theorem.
5. What is the determinant of a 2×2 matrix whose rank is 1 ?
6. What is the normal form of the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{pmatrix}$?
7. Define eigen value of a matrix.
8. Define divergence of a vector field.
9. Define gradient of a function.
10. Define the derivative of a vector function.
11. Define a smooth curve.
12. State Gauss's divergence theorem.

($12 \times 1 = 12$ marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Solve the initial value problem $y' = 3y, y(0) = 5.7$.

14. Find an integrating factor for $2\cosh x \cos y dx = \sinh x \sin y dy$ and solve it.

15. Find the angles of the triangle with vertices $(0,0,0), (1,2,3), (4,-1,3)$.

16. Find x, y, z, t where $3\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2t \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+t & 3 \end{pmatrix}$.

17. Solve the system :

$$x - 3y = 4$$
$$-2x + 6y = -8.$$

18. Show that circle of radius a has curvature $\frac{1}{a}$.

19. Find a unit normal vector n of the cone of revolution $z^2 = 4(x^2 + y^2)$ at the point $(1, 0, 2)$.

20. Find the directional derivative of $f = x^2 + y^2 - z$ at $(1, 1, -2)$ in the direction of $(1, 1, 2)$.

21. Show that $\text{curl}(u + v) = \text{curl}\, u + \text{curl}\, v$.

22. Show that $\text{div}\, kv = k\,\text{div}\,v$.

23. Show that $\int_{(0,\pi)}^{\left(3,\frac{\pi}{2}\right)} e^x (\cos y\, dx - \sin y\, dy)$ is path independent.

24. Write the formula for finding the area of a plane region as a line integral over the boundary.

(9 × 2 = 18 marks)

Part C (Short Essay)

Answer any **six** *questions.*
Each question carries 5 marks.

25. Show that the form under integral sign is exact in the plane and evaluate the integral

$$\int_{(-1,-1)}^{(1,1)} e^{-x^2-y^2}\,(xdx + ydy).$$

26. Solve $2x \tan y\, dx + \sec^2 y dy = 0$.

27. Find the minimal polynomial $m(t)$ of $A = \begin{pmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{pmatrix}$.

28. Let $A = \begin{pmatrix} 3 & -4 \\ 2 & -6 \end{pmatrix}$. Find all eigen values and corresponding eigen vectors. Find matrices P and D such that P is non-singular and $D = P^{-1}AP$ is diagonal.

29. Let L be the linear transformation on R^2 that reflects each point P across the line $y = kx$, where $k > 0$.

    (a) Show that $v_1 = (k, 1)$ and $v_2 = (1, -k)$ are eigenvectors of L.

    (b) Show that L is diagonalizable, and find a diagonal representation D.

30. Find the straight line L_1 through the point $P:(1,3)$ in the xy-plane and perpendicular to the straight line $L_2 : x - 2y + 2 = 0$.

31. Evaluate the double integral $\iint_R y^2\, dxdy$ where R is the region bounded by the unit circle in the first quadrant.

32. Solve $2x \tan y dx + \sec^2 y dy = 0$.

33. Verify Greens theorem in the plane for $F = \left[-y^3, x^3\right]$ and the region is the circle $x^2 + y^2 = 25$.

(6 × 5 = 30 marks)

Turn over

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Test for consistency and solve the following system :

(a) $$\begin{aligned} x_1 + x_2 - 2x_3 + 4x_4 &= 5 \\ 2x_1 + 2x_2 - 3x_3 + x_4 &= 3 \\ 3x_1 + 3x_2 - 4x_3 - 2x_4 &= 1. \end{aligned}$$

(b) $$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y - z &= -4 \\ 3x - 2y - z &= 5. \end{aligned}$$

35. Solve :

(a) $2 \sin\left(y^2\right) dx + xy \cos\left(y^2\right) dy = 0,\ y(2) = \sqrt{\frac{\pi}{2}}.$

(b) Find the angle between $x - y = 1$ and $x - 2y = -1$.

36. Evaluate $\iint_S (7xi - zk) \cdot n dA$ over the sphere $S: x^2 + y^2 + z^2 = 4$ by

(a) Divergence theorem.

(b) Directly.

(2 × 10 = 20 marks)

D 91713-A (Pages : 4) Name.....................................

Reg. No.....................................

THIRD SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION NOVEMBER 2020

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

#

(Multiple Choice Questions for SDE Candidates)

1. $\int \frac{\ln x}{x} dx =$ ————.

   (A) $\frac{1}{x^2} + c.$

   (B) $\frac{(\ln x)^2}{2} + c.$

   (C) $\frac{\ln x}{2} + c.$

   (D) $\frac{1}{x} - \ln x + c$

2. $\int \tan x \, dx =$ ————.

   (A) $\ln|\cos x| + c.$

   (B) $\ln|\sec x| + c.$

   (C) $\sec^2 x + c.$

   (D) $\ln|\sin x| + c.$

3. $\lim_{x \to 1} \frac{1 - x}{\ln x} =$ ————.

   (A) $-1.$

   (B) $+1.$

   (C) $0.$

   (D) $2.$

4. Range of tan hx is ————.

   (A) $[-1,1].$

   (B) $(-1,1).$

   (C) $(-\infty,\infty).$

   (D) $(0,\infty).$

5. $a_n = \frac{2n+1}{3n+5}$. The $\lim_{n \to \infty} a_n =$ ————.

   (A) $\frac{1}{5}.$

   (B) $\frac{2}{5}.$

   (C) $\frac{2}{3}.$

   (D) $\frac{3}{2}.$

6. The sequence $\left\{\frac{1}{n}\right\}$ is :

   (A) Diverges.

   (B) Increasing.

   (C) Decreasing.

   (D) None of these.

7. $\lim_{n \to \infty} a_n = 2$, $\lim_{n \to \infty} b_n = 4$, then $\lim_{n \to \infty} (a_n + b_n) =$ ————.

(A) 6.

(B) 10.

(C) 8.

(D) 16.

8. $\lim_{n \to \infty} x^{1/n} (x > 0) =$ ————.

(A) 0.

(B) 1.

(C) ∞.

(D) None of these.

9. $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n =$ ————.

(A) 1.

(B) 0.

(C) e.

(D) $\frac{1}{e}$.

10. Fourth term of the sequence $\left\{\frac{(-1)^{n+1}}{2^{n+1}}\right\}$ is :

(A) $\frac{1}{2^5}$.

(B) $\frac{-1}{2^5}$.

(C) $\frac{1}{2^4}$.

(D) $\frac{-1}{2^4}$.

11. The series $\sum_{n=1}^{\infty} a_n$ converges, then :

(A) $\lim_{n \to \infty} a_n = c$, a constant.

(B) $\lim_{n \to \infty} a_n = 0$.

(C) $\lim_{n \to \infty} a_n = \infty$.

(D) $\lim_{n \to \infty} a_n$ does not exist.

12. Let $a_n > 0$ and $b_n > 0$ and $\lim_{n \to \infty} \frac{a_n}{b_n} = l$. Then $\sum a_n$ and $\sum b_n$ converges if :

(A) $l = 0$.

(B) $l > 0$.

(C) $l = \infty$.

(D) None of these.

13. $\sum a_n$ be a series such with $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = l$. Then :

(A) $\sum a_n$ converges if $l > 1$.

(B) $\sum a_n$ diverges if $l > 0$.

(C) $\sum a_n$ converges if $l < 1$.

(D) $\sum a_n$ diverges if $l = 0$.

14. $\sum a_n$ be a series such with $a_n \geq 0 \lim\limits_{n \to \infty} (a_n)^{1/n} = l$. Then :

(A) $\sum a_n$ converges if $l < 1$.
(B) $\sum a_n$ diverges if $l < 1$.
(C) $\sum a_n$ converges if $l = 1$.
(D) $\sum a_n$ diverges if $l = 1$.

15. If the series $\sum a_n (x-c)^n$ converges for $x = k$. Then converges in :

(A) $|r - c| < k$.
(B) $|r - c| \leq k$.
(C) $|r - c| > k$.
(D) $|r - c| \geq k$.

16. The series $\sum\limits_{n=0}^{\infty} x^n$ is :

(A) Converges absolutely for $|x| < 1$.
(B) Converges for $|x| > 1$.
(C) Has radius of converges $\frac{1}{2}$.
(D) None of these.

17. The vertex of the parabola whose focus is (1, –1) and whose directrix passes through (3, 3) is :

(A) (–2, 1).
(B) (2, 1).
(C) (1, 2).
(D) (–1, 2).

18. Equation of the hyperbola with foci $(0, \pm\sqrt{2})$ and asymptote $y = x$:

(A) $x^2 - y^2 = 1$.
(B) $\frac{x^2}{4} - \frac{y^2}{2} = 1$.
(C) $y^2 - x^2 = 1$.
(D) $\frac{y^2}{4} - \frac{x^2}{2} = 1$.

19. Eccentricity of the hyperbola $9x^2 - 16y^2 = 144$ is :

(A) $\frac{7}{5}$.
(B) $\frac{5}{4}$.
(C) $\frac{6}{5}$.
(D) $\frac{7}{4}$.

20. The equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$ represent a ellipse if :

(A) (A > 0 and C > 0).
(B) (A > 0 and C < 0).
(C) (A < 0 and C > 0).
(D) (A ≠ 0, C ≠ 0).

D 91713 (Pages : 4) Name.......................................

Reg. No..................................

THIRD SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION NOVEMBER 2020

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Evaluate $\int_{0}^{\pi/6} \tan 2x \, dx$.

2. Define an alternating series.

3. Find the vertices of the hyperbola $\frac{y^2}{4} - \frac{x^2}{5} = 1$.

4. Find the Taylor polynomial of order zero generated by $f(x) = \sin x$ at $a = \frac{\pi}{4}$.

5. Evaluate $\frac{d}{dt}\left(\tan h\sqrt{1+t^2}\right)$.

6. Examine whether $\sum_{n=1}^{\infty} n^2$ converges or diverges.

7. Find the directive of the parabola $y^2 = 10x$.

8. Define absolute convergence.

9. Find y if $\ln y = 3t + 5$.

10. Find the parametric equation of the circle $x^2 + y^2 = a^2$.

Turn over

11. Examine whether $x^2 + xy + y^2 - 1 = 0$ represents a parabola, ellipse or hyperbola.

12. Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. For what values of x do the series $\sum_{n=0}^{\infty} n!\, x^n$ converges.

14. Evaluate $\frac{d}{dx} \ln_{10}(3x+1)$.

15. Evaluate $\int \frac{\log_2 x}{x}\, dx$.

16. Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $a = 2$.

17. Find $\frac{dy}{dx}$ if $y = x^x, x > 0$.

18. Graph the set of points whose polar co-ordinates satisfy the conditions $1 \le r \le 2$ and $0 \le 0 \le \frac{\pi}{2}$.

19. Examine whether the series :

$$5 + \frac{2}{3} + 1 + \frac{1}{7} + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \ldots + \frac{1}{k!} + \ldots$$ converges.

20. Evaluate $\int_{0}^{\ln 2} 4e^x \sin h x \; dx$.

21. Show that ln x grows slaver than x as $x \to \infty$.

22. Examine whether $\sum_{n=1}^{\infty} (-1)^{n+1}$ converges or diverges.

23. Evaluate $\lim_{x \to 0} \frac{3x - \sin x}{x}$.

24. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3+2\sin\theta} d\theta$.

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. The standard parametrization of the circle of radius 1 centered at the point (0, 1) in the xy-plane is $x = \cos t, y = 1 + \sin t, 0 \le t \le 2\pi$. Use the parametrization to find the area of the surface swept out by revolving the circle about the x-axis.

26. Find the centroid of the first quadrant of the astroid $x = \cos^3 t, y = \sin^3 t, 0 \le t \le 2\pi$.

27. Show that $\lim_{x \to 0^+} (1+x)^{\frac{1}{x}} = e$.

28. Using Integral test show that the p-series :

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \ldots + \frac{1}{n^p} + \ldots \text{ converges if } p > 1 \text{ and diverges if } p \le 1.$$

29. Find the Taylor polynomial generated by $f(x) = \cos x$ at $x = 0$.

30. Find the length of the Cardioid $r = 1 - \cos\theta$.

31. Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

32. Graph the curve $r^2 = 4\cos\theta$.

33. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34 Find the area of the region in the plane enclosed by the Cardioid $r = 2(1+\cos\theta)$.

35 Show that the Maclaurin's series for cos x converges to cos x for every value of x.

36 Using Integral test examine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

(2 × 10 = 20 marks)