C 4389-A (Pages : 3) Name..................................

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MEC 2C 02—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 15	Maximum : 15 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MEC 2C 02—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. A system with m equations and n variables has at most ———————— basic solutions.

   (A) $n - m$.
   (B) $m - n$.
   (C) Cm.
   (D) None of these.

2. A basic feasible solution is a basic solution whose variables are ————————.

   (A) Feasible.
   (B) Negative.
   (C) Non-negative.
   (D) None.

3. The line joining (0, 0) to (100,100) in a Lorenz Curve is called :

   (A) Line of equal distribution.
   (B) Sloping line.
   (C) Line of perfect inequality.
   (D) None of these.

4. The divergence between Lorenz curve and line of perfect equality can be measured by :

   (A) Gini co-efficient.
   (B) Co-efficient of variation.
   (C) Both.
   (D) None.

5. In LPP, simplex method was developed by :

   (A) Koopman.
   (B) G.B. Dantzig.
   (C) Leonteif.
   (D) None of these.

6. Any non negative value of (x_1, x_2) is a constraints :

   (A) Critical region.
   (B) Feasible region.
   (C) Optimal solution.
   (D) None.

7. The optimal solution any LPP corresponds to one of the ———————— of the feasible region.

   (A) Turning points.
   (B) Corner points.
   (C) Maximum point.
   (D) None.

8. In LPP we deal with ———————— objectives.

   (A) Many.
   (B) Two.
   (C) Three.
   (D) Four.

9. The effect of changes in the co-efficient in the optimum value of the objective function can be studied through a technique called :

   (A) Simplex method.
   (B) Sentitivity analysis.
   (C) Assignment problem.
   (D) None of these.

10. Dual of the dual is called :

   (A) Simplex.
   (B) Primal.
   (C) Both.
   (D) None.

11. When both players use their optimal minimax strategies the resulting expected pay off is called :

   (A) Game.
   (B) Strategy.
   (C) Both.
   (D) None.

12. In a game, minimax value is ———————— maximin value.

   (A) Greater than.
   (B) Geater than or equal to.
   (C) Less than.
   (D) None.

13. Saddle point is also called :

   (A) Critical point.
   (B) Equilitarian point.
   (C) Both.
   (D) None.

14. The input-output analysis technique was propounded by :

   (A) Van Neumann.
   (B) W.W. Leontief.
   (C) C.F. Christ.
   (D) None.

15. An input-output model in which some of the production is consumed by external bodies is called :

   (A) Open.
   (B) Closed.
   (C) Both (A) and (B).
   (D) None.

C 4389 (Pages : 2) Name..

Reg. No....................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2021

Mathematics

MEC 2C 02—MATHEMATICAL ECONOMICS

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Define a Lorenz curve.
2. What is meant by Line of best fit ?
3. Explain Closed input output model.
4. What is an Endogenous variable ?
5. State the Chain rule.
6. What is an Objective function ?
7. State the Law of diminishing marginal productivity.
8. Define Jacobian derivative.
9. What is meant by Unconstrained optimization ?
10. What is meant by a Saddle point ?
11. What is meant by Technological coefficient matrix ?
12. Explain the term Nondegenarate constraint qualification.

(8 × 3 = 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Explain the measurement of income inequality using Lorenz curve.

14. Explain the method of least squares.

15. Examine whether the input-output system with the following co-efficient matrix is feasible :

$$\begin{bmatrix} 0.5 & 0.6 \\ 0.3 & 0.7 \end{bmatrix}.$$

16. Derive the profit maximizing conditions for a discriminating monopolist selling in two markets.

17. Explain the Hessian Matrix.

18. Explain the limitations of input-output analysis

19. Describe the Kuhn-Tucker formulation for a constrained minimization problem.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Explain the conditions for profit maximization of a firm. If the cost function of a firm is given by $C = 25x + 10000$ and the demand function is given as $p = -1/5x + 500$, find the profit maximizing level of output.

21. Explain the basic assumptions of input-output analysis. The final demand of two sectors is given as X_2a = Rs. 20 crores and X_2b = Rs. 15 crores. The co-efficient matrix is given as

$$C = \begin{bmatrix} 0.4 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}.$$ Find the levels of output X_1 and X_2.

(1 × 11 = 11 marks)

C 4388-B

(Pages : 3)

Name.......................................

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MAT 2C 02 MATHEMATICS—2

(2020 Admissions)

Time : Two Hours

Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Prove that $\cosh^2 x - \sinh^2 x = 1$.

2. Find the Cartesian form of the polar equation $r = \frac{8}{1-2\cos\theta}$.

3. Find the slope of the line tangent to the graph of $r = 3\cos^2 2\theta$ at $\theta = \pi/6$.

4. Evaluate $\int \sinh^2 x dx$.

5. Show that $\lim_{n\to\infty} \frac{2n}{n^2+1} = 0$.

6. Test the convergence of the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{8}} + \frac{1}{\sqrt{16}} \dots$

7. Compute $\|\cos x\|$ in $C[0,2\pi]$.

8. Examine whether the set of vectors $u_1 = \langle 1,2,3\rangle, u_2 = \langle 2,4,3\rangle$, and $u_3 = \langle 3,2,1\rangle$ is linearly independent or not.

9. Find the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}$.

10. Find the determinant of the matrix $C = \begin{bmatrix} -1 & 2 & 9 \\ 2 & -4 & -18 \\ 5 & 7 & 27 \end{bmatrix}$.

11. Show that $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is an orthogonal matrix.

12. Find the eigen values of the matrix $A = \begin{vmatrix} 10 & 3 \\ 4 & 6 \end{vmatrix}$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1, 0 \le x \le 1$.

14. Find the equation of the tangent line when $t = 1$ for the curve $x = t^4 + 2\sqrt{t}, y = \sin(t\pi)$.

15. Find the length of the perimeter of the cardioid $r = a(1 - \cos\theta)$.

16. Use the Trapezoidal rule with $n = 4$ to estimate $\int_1^2 x^2 dx$. Compare the estimate with the exact value of the integral.

17. Using Maclaurin's series expand $\tan^{-1} x$. Hence deduce the Gregory series $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

18. Show that the set B = {(1, 2, 1), (2, 1, 0), (1, –1, 2)} is a basis for R^3.

19. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Evaluate $\int_1^{\infty} \frac{\ln x}{x^2}\,dx$, if it exists.

(b) Find the area of the region shared by the cardioids $r = 2(1+\cos\theta)$ and $r = 2(1-\cos\theta)$.

21. (a) Solve :

$$x_1 + x_2 + x_3 + x_4 = 0$$
$$x_1 + 3x_2 + 2x_3 + 4x_4 = 0$$
$$2x_1 + x_3 - x_4 = 0.$$

(b) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & -6 \\ 2 & 2 \end{bmatrix}$.

(1 × 11 = 11 marks)

C 4388 (Pages : 3) Name.....................................

Reg. No.....................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MAT 2C 02—MATHEMATICS—2

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks
All questions can be attended.
Overall Ceiling 24.

1. Compute the derivative of $\sqrt{x}$ using inverse function rule. Evaluate the derivative at $x = 2$.

2. Convert the relation $r = 1 + 2\cos\theta$ to Cartesian co-ordinates.

3. Compute $\int \cosh^2 x \, dx$.

4. Find $\frac{d}{dx} \cosh^{-1} \sqrt{x^2 + 1}, x \neq 0$.

5. Find $\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$.

6. Show that $\int_0^{\infty} \frac{\sin x}{\left(1 + x^2\right)} dx$ converges.

7. A bouncing ball loses half of its energy on each bounce. The height reached on each bounce is proportional to the energy. Suppose that the ball is dropped vertically from a height of one meter. How far does it travel ?

8. State Ratio comparison test and show that $\sum_{i=1}^{\infty} \frac{2}{4+i}$ diverges.

9. Prove that the vectors $w_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $w_2 = \left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and $w_3 = \left(0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ are orthonormal vectors.

10. Define basis of a vector space. Give a basis for vector space P_n of all polynomial of degree less than or equal to n.

11. Find the inverse of $A = \begin{pmatrix} 1 & 8 \\ 2 & 10 \end{pmatrix}$.

12. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 3 & 4 \\ -1 & 7 \end{pmatrix}$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Let $f(x) = x^2 + 2x + 3$. Restrict f to a suitable interval so that it has an inverse. Find the inverse function and sketch its graph.

14. Find the length of the graph of $f(x) = (x-1)^{3/2} + 2$ on $[0, 2]$.

15. State root test and test the convergence for the series $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$.

16. For which x does the series $\sum_{n=0}^{\infty} \frac{4^n}{\sqrt{2n+5}} (x+5)^n$ converge.

17. Let $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 2)$ and $u_3 = (1, 1, 0)$ be basis of $\mathbb{R}^3$. Using Gram Schimdt process find an orthonormal basis of $\mathbb{R}^3$.

18. Compute A^m for $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$.

19. Identify the conic whose equation is $2x^2 + 4xy - y^2 = 1$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 marks.

20. (a) Find the area of the surface obtained by revolving the graph $y = x^2$ about the y-axis for $1 \le x \le 2$.

(b) Determine whether the set of vectors $u_1 = (1, 2, 3)$, $u_2 = (1, 0, 1)$ and $u_3 = (1, -1, 5)$ is linearly dependent or linearly independent.

21. (a) Find the terms through cubic order in the Taylor series for $\dfrac{1}{1 + x^2}$ at $x_0 = 1$.

(b) Find an LU factorization of $A = \begin{pmatrix} -1 & 2 & -4 \\ 2 & -5 & 10 \\ 3 & 1 & 6 \end{pmatrix}$.

(1 × 11 = 11 marks)

C 4387-A (Pages : 6) Name..

Reg. No......................................

SECOND SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE – I

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 2B 02—CALCULUS OF SINGLE VARIABLE – I

(Multiple Choice Questions for SDE Candidates)

1. The domain of $f(x) = \sqrt{x^2 - 1}$ is :

   (A) $[1,\infty)$.
   (B) $(-\infty,-1]\cup[1,\infty)$.
   (C) $(-\infty,\infty)$.
   (D) $(0,\infty)$.

2. $\lim_{x\to 0}\frac{\tan(x)}{x}$ is :

   (A) 0.
   (B) 1.
   (C) π.
   (D) Limit does not exists.

3. Which of the following functions are continuous for all real number x ?

   (A) $\tan(x)$.
   (B) $\frac{1}{x}$.
   (C) $\sec(x)$.
   (D) $\exp x$.

4. $\lim_{x\to a} f(x) = L$ if and only if :

   (A) $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$.
   (B) $f(a) = L$.
   (C) $\lim_{x\to a^-} f(x) = L$
   (D) None of the above.

5. On $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, $f(x) = \cos(x)$ takes on :

   (A) A maximum value of 1 (once) and a minimum value ol 0 (twice).
   (B) A maximum value of 1 (once) and no minimum value.
   (C) A minimum value of 0 (twice) and no maximum value.
   (D) A maximum value of 1 (once) and a minimum value of 0 (once).

6. Consider the function :

$$f(x) = \begin{cases} x+1 & -1 < x < 0 \\ 0 & x = 0 \\ x-1 & 0 < x < 1. \end{cases}$$

Then which of the following statements is NOT true ?

(A) f is continuous at every point of [-1, 1], except at $x = 0$.

(B) f has a non-removable discontinuity at $x = 0$.

(C) f has neither a highest nor a lowest point on [-1, 1].

(D) f has the highest value 1 and the lowest value -1 on [-1, 1].

7. What are the critical points of f when $f'(x) = (x-1)(x-2)$?

(A) 0,1 and 2.

(B) -1 and -2.

(C) 1 and 2.

(D) No critical points.

8. The value or values of c that satisfy the equation $\frac{f(b)-f(a)}{b-a} = f'(c)$ in the conclusion of Mean Value Theorem for the function $f(x) = x^2 + 2x - 1$ and the interval [0,1] is :

(A) 1.

(B) $\frac{1}{2}$.

(C) $\frac{1}{3}$.

(D) $\frac{1}{4}$.

9. The tangent at the point of inflection is called ————.

(A) Inflectional tangent.

(B) Vertical tangent.

(C) Asymptote.

(D) None of these.

10. $\lim\limits_{x\to\infty} \dfrac{5x^2+8x-3}{3x^2+2}$ ———.

(A) $\dfrac{5}{3}$.

(B) 5.

(C) 0.

(D) ∞.

11. $\lim\limits_{x\to 0} \dfrac{\sin(3x)}{\sin(4x)}$:

(A) 1.

(B) $\dfrac{4}{3}$.

(C) $\dfrac{3}{4}$.

(D) 0.

12. $\lim\limits_{x\to 0} \dfrac{|x|}{x}$:

(A) 1.

(B) 0.

(C) Does not exist.

(D) −1.

13. Find the maximum value of $f(x) = x - 2lnx$ on the interval [1,4] :

(A) 1.

(B) 3 − 2ln3.

(C) 2 −2ln2.

(D) 2.

14. The linearization of a function $f(x) = \cos(2x)$ at the $x = \dfrac{1}{2}$.

(A) $y = \cos(1) - 2\sin(1)\left(x - \dfrac{1}{2}\right)$.

(B) $y = \cos(2) - 2\sin(2)\left(x - \dfrac{1}{2}\right)$.

(C) $y = \dfrac{1}{2} - 2\sin(1)\left(x - \dfrac{1}{2}\right)$.

(D) $y = \cos(1) + \sin(1)\left(x - \dfrac{1}{2}\right)$.

15. If $y = 12\sqrt{x^3} + 8$, then $\frac{dy}{dx}$ is :

(A) 18x.

(B) $18\sqrt{x^5}$.

(C) 18x.

(D) $18\sqrt{x}$.

16. If $y = \sin 3x$, then dy is ————.

(A) $dy = (\cos 3x)dx$.

(B) $dy = (-3\cos 3x)dx$.

(C) $dy = (3\cos 3x)dx$.

(D) $dy = (3\sin 3x)dx$.

17. $d\left(\frac{x}{x+1}\right) =$ ————.

(A) $\frac{dx}{x+1}$.

(B) $\frac{x\,dx}{x+1}$.

(C) $\frac{x\,dx}{(x+1)^2}$.

(D) $\frac{dx}{(x+1)^2}$.

18. The radius r of a circle increases from $r_0 = 10$ m to 10.1 m. Estimate the increase in the circle's area A by calculating dA.

(A) $dA = 2\pi\, m^2$.

(B) $dA = -2\pi\, m^2$.

(C) $dA = \pi\, m^2$.

(D) $dA = -\pi\, m^2$.

19. Define norm of a partition :

(A) The norm of a partition P is the first subinterval length.

(B) The norm of a partition P is the average of partition's subinterval length.

(C) The norm of a partition P is the partition's shortest subinterval length.

(D) The norm of a partition P is the partition's longest subinterval length.

20. If $f(x) \geq g(x)$ on $\{a,b\}$, then :

(A) $\int_a^b f(x)dx \geq \int_a^b g(x)dx.$

(B) $\int_a^b f(x)dx \leq \int_a^b g(x)dx.$

(C) $\int_a^b f(x)dx = \int_a^b g(x)dx.$

(D) $\int_a^b f(x)dx = -\int_a^b g(x)dx.$

C 4387 (Pages : 3) Name...

Reg. No.....................................

SECOND SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE – I

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 30.

1. Find two functions f and g such that $F = g \circ f$ if $F(x) = (x+2)^4$.

2. Let $f(x) = \begin{cases} -x+3 & \text{if } x < 2 \\ \sqrt{x-2+1} & \text{if } x \geq 2 \end{cases}$

   Find $\lim_{x \to 2} f(x)$ if it exists.

3. Find the values of x for which the function $f(x) = x^8 - 3x^4 + x + 4 + \dfrac{x+1}{(x+1)(x-2)}$ is continuous.

4. Find $\lim_{x \to o} \dfrac{\tan x}{x}$.

5. Find the instantaneous rate of change of $f(x) = 2x^2 + 1$ at $x = 1$.

6. If $f(x) = 2x^3 - 4x$. Find $f'(-2)$ and $f'(0)$.

7. Find the extreme values $f(x) = 3x^4 - 4x^3 - 8$ on $[-1, 2]$.

8. Determine where the graph of $f(x) = x^3 - 6x$ is concave upward and where it is concave downward.

9. Find $\lim_{x \to -1} \dfrac{1}{x+1}$.

10. Find the horizontal asymptote of the graph of $f(x) = \dfrac{1}{x-1}$.

Turn over

11. Find $\int \frac{\sin t}{\cos^2 t}\, dt$

12. Find $\int \frac{1}{x \log x}\, dx$.

13. Given that $\int_2^2 f(x)dx = 3$ and $\int_0^2 f(x)\, dx = 2$, evaluate $\int_2^0 f(x)\, dx$.

14. Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = -x$.

15. Find the volume of the solid obtained by revolving the region bounded by $y = x^3$, $y = 8$ and $x = 0$ about the y-axis.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 marks.
All questions can be attended.
Overall Ceiling 30.

16. Show that the function $f(x) = |x|$ is differentiable everywhere except at 0.

17. Show that if the function f is differentiable at a, then f is continuous at a.

18. Verify that the function $f(x) = x^2 + 1$ satisfies the hypothesis of the mean value theorem on [0, 2] and find all values of c that satisfy the conclusion of the theorem.

19. Find the relation extrema if any of the function $h(t) = \frac{1}{3}t^3 - 2t^2 - 5t - 10$.

20. The velocity function of a car moving along a straight road is given by $v(t) = t - 20$, for $0 \le t \le 40$, where $r(t)$ is measured in feet per second and t in seconds. Show that at $t = 40$, the car will be in the same position as it was initially.

21. (a) State mean value theorem for integrals.

    (b) Verify mean value theorem for $f(x) = x^2$ on [1, 4].

22. (a) Use differentials to obtain an approximation of the arc length of the graph of $y = 2x^2 + x$ from P (1,3) to Q (1.1, 3.52).

(b) Find the work done in lifting a 50 – lb sack of potatoes to a weight of 4 ft above the ground.

23. Find the length of the graph of $f(x) = \frac{1}{3}x^3 + \frac{1}{4x}$ on the interval [1, 3].

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Find the slope and an equation of the tangent line to the graph $f(x) = x^2$ at the point (1, 1).

(b) Suppose that the total cost in dollars incurred per weak by a company in manufacturing x refrigerators is given by the total cost function $c(x) = -0.2x^2 + 200x + 9000$, $0 \le x \le 400$.

(i) What is the cost incurred in manufacturing the 201 st refrigerator ?

(ii) Find the rate of change of c with respect to x when $x = 200$.

25. A man has 100 ft of fencing to enclose a rectangular garden. Find the dimensions of the garden of largest area he can have if he uses all of the fencing.

26. (a) Estimate $\int_0^t e^{-\sqrt{x}}\,dx$ using the property of definite integral.

(b) Use the geometric interpretation of the integral to evaluate $\int_{-1}^{2} |x-1|\,dx$ by making a sketch of f.

27. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

(2 × 10 = 20 marks)

C 4163-A (Pages : 4) Name....................................

Reg. No..................................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

ME 2C 02—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

ME 2C 02—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. ———— is a measure of inequality in variables.

(A) Binomial distribution. (B) Poisson distribution.

(C) Pareto distribution. (D) None of these.

2. While drawing Lorenz curve, cumulative percentage of income is taken along the :

(A) X-axis. (B) Y-axis.

(C) XY plane. (D) None of these.

3. The farther the Lorenz curve from the line of equal distribution, the ———— inequality in income.

(A) Lesser (B) Poorer.

(C) Greater. (D) Moderate.

4. LPP the transportation problem was contributed by :

(A) Leonteif. (B) Koop man.

(C) G.B. Dantzig. (D) None.

5. Which of the following is an assumption of LP ?

(A) Certainty. (B) Divisibility.

(C) Additivity. (D) All the above.

6. LP is a quantitative technique of decision making using ———— constraints.

(A) Inequality. (B) Equality.

(C) Both (A) and (B) (D) None.

7. One of the limitations of LPP is to satisfy the assumption of ———— of objective function and constraints.

(A) Certainty. (B) Continuity.

(C) Linearity. (D) None.

8. If no feasible solution of the problem exists, then that LPP is said to be :

(A) Unbounded. (B) Bounded.

(C) Infeasible. (D) Feasible.

9. A system has 3 equations with 3 variables. If it is solvable, such a solution is called :

(A) Optimal solution. (B) Basic solution.

(C) Multiple solution. (D) None.

10. A system with m equations and n variables has at most ———— basic solutions.

(A) n-m. (B) m-n.

(C) cCm. (D) None of these.

11. If the primal problem is a maximisation problem, then the dual problem is a ———— problem.

(A) Minimisation. (B) Transportation.

(C) Assignment. (D) None.

12. The method of solving LPP with greater than or equal to type constraints is :

(A) Two phase method. (B) M method.

(C) Both (A) and (B). (D) None.

13. Game theory is really the ————.

(A) Conflict. (B) Science of conflict.

(C) Competition. (D) None.

14. When there is no saddle point in a game, it will be the case of ?

(A) Pure strategy. (B) No strategy.

(C) Mixed strategy. (D) None of these.

15. An input-output model in which some of the production is consumed by external bodies is called :

(A) Open. (B) Closed.

(C) Both (A) and (B). (D) None.

16. When the Lorenz Curve coincides with the line of perfect equality, it indicates ?

(A) No inequality.
(B) 100% inequality.
(C) 50% inequality.
(D) None of the above.

17. For a LPP, if there is a tie in the net evaluation row of the simplex table corresponding to an iteration :

(A) The tie should be broken before proceeding further.
(B) We can choose any one and proceed.
(C) It means something has gone wrong.
(D) The solution may cycle round.

18. In linear programming the number of constraints in the dual of a given primal problem is :

(A) The number of variables in the primal.
(B) The number of constraints in the primal.
(C) The number of constants in the primal.
(D) None of the above.

19. A constant sum game is one in which :

(A) The total gain is constantly changing.
(B) The total gain is indeterminate.
(C) The total gain is fixed.
(D) None of the above.

20. The pay off matrix in a game refers to :

(A) The total pay-off in a game.
(B) The pay-off to one player in a game.
(C) The payments made by players in a co-operative game.
(D) None of the above.

C 4163 (Pages : 4) Name.......................................

Reg. No...................................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

ME 2C 02—MATHEMATICAL ECONOMICS

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define income inequality.
2. Write any two merits of Gini coefficient.
3. What are constraints ?
4. Define feasible region.
5. Define a slack variable.
6. What is a dual problem ?
7. What is a shadow price ?
8. What is a zero sum game ?
9. What is saddle point ?
10. Define Leontieff matrix.
11. State two limitations of input-out model.
12. What is the input-output technique ?

(12 × 1 = 12 marks)

Part B

Answer any **six** *questions in two* **or** *three sentences.*
Each question carries 3 *marks.*

13. What is a Lorenz Curve ? Write any two limitations of Lorenz curve.
14. Write short note on Income tax and wealth tax.

15. Write the dual of the LPP : Maximize

$\pi = 120x_1 + 360x_2$

subject to

$7x_1 + 2x_2 \leq 28$
$3x_1 + 9x_2 \leq 36$
$6x_1 + 4x_2 \leq 48$
$x_1, x_2 \geq 0$

16. Discuss theory of games.
17. Explain the interrelationship between linear programming and game theory.
18. Define value of a game.
19. What is input-output co-efficients ? What conditions may be filled if they are to be stable through time ?
20. What do you understand by Leontief Inverse Matrix and what does it show ?
21. Discuss the significance of input-output table.

(6 × 3 = 18 marks)

Part C

Answer any **six** *questions from the following.*
Each question carries 5 marks.

22. A manufacturer makes two products x_1 and x_2. The first requires 5 hours for processing, 3 hours for assembling, and 4 hours for packaging. The second requires 2 hours for processing, 12 hours for assembling, and 8 hours for packaging. The plant has 40 hours available for processing, 60 for assembling, and 48 for packaging. The profit margin for x_1 is Rs. 7 ; for x_2 it is Rs. 21. Express the data in equations and inequalities necessary to determine the output mix that will maximize profits.
23. Solve graphically : Maximize $\pi = 8x_1 + 6x_2$ subject to $2x_1 + 5x_2 \leq 40$

$3x_1 + 3x_2 \leq 30$
$8x_1 + 4x_2 \leq 64$
$x_1, x_2 \geq 0$

24. Solve graphically : Minimize $c = 6y_1 + 3y_2$

subject to $y_1 + 2y_2 \geq 14$

$y_1 + y_2 \geq 12$
$3y_1 + y_2 \geq 18$
$y_1, y_2 \geq 0$

25. Solve the game with pay off matrix

$$M = \begin{bmatrix} 1 & 4 \\ 5 & 3 \end{bmatrix}$$

26. Solve the game graphically with pay off matrix

$$M = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix}$$

27. Illustrate the concept of Dominance with an example.

28. Write any five limitations of input-output, analysis.

29. In a two industry economy, it is known that industry I uses 10 paise of its own product and 60 paise of commodity II to produce a rupee worth of commodity I ; industry II uses non of its own product but uses 50 paise of commodity I in producing a rupees worth of commodity II and the open sector demands are Rs. 1,000 for commodity I and Rs. 2,000 for commodity II. Find the technology matrix.

30. Given the demand $F_1 = 11$ and $F_2 = 12$. Find the solution output mix for the two industries, given the input matrix

$$A = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}$$

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions from the following.*
Each question carries 10 *marks.*

31. Solve using simplex method : Maximize $\pi = 56x_1 + 24x_2 + x_3$

subject to $4x_1 + 2x_2 + 3x_3 \le 240$

$8x_1 + 2x_2 + x_3 \le 120$

$x_1, x_2, x_3 \ge 0$

32. Solve using simplex method :

Minimize $c = 14y_1 + 40y_2 + 18y_3$

subject to

$2y_1 + 5y_2 + y_3 \ge 50$

$y_1 + 5y_2 + 3y_3 \ge 30$

$y_1, y_2, y_3 \ge 0$

33. Determine the optimum strategies of the game for the following payoff matrix

$$M = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}.$$

34. In a three sector economy, the input co-efficient matrix and final demand vector are as given below :

$$A = \begin{bmatrix} 0.3 & 0.2 & 0.3 \\ 0.1 & 0.3 & 0.4 \\ 0.2 & 0.3 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 500 \\ 700 \\ 600 \end{bmatrix}$$

Find the sectoral output X_1, X_2 and X_3 using Cramers rule.

(2 × 10 = 20 marks)

C 4162 (Pages : 4) Name.......................................

Reg. No...................................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type Questions)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Find $\int a^x \, dx$.

2. Define the partial derivative of $f(x, y)$ with respect to x at (x_0, y_0).

3. If f is continuous on $[a, \infty)$, then $\lim_{b \to \infty} \int_a^b f(x)\, dx =$ ....

4. If a series $\sum a_n$ an converges, then $\lim a_n =$ ....

5. Find the n^{th} term of the sequence $-2, 2, -2, 2, \ldots\ldots$

6. Find the domain of the function $z = \sin xy$.

7. The polar form of the line $y = 2$ is ....

8. $\frac{d}{dx} \cosh x =$ ....

9. If $f(x, y) = 2x^2 y$ then find $\frac{\partial^2 f}{\partial x \partial y}$.

10. Give an example of conditionally converging series.

11. State Sandwich theorem for sequences.

12. Write the transformation equations for Cartesian co-ordinates to spherical polar co-ordinates.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Evaluate $\int_0^{\log 2} 4e^x \sinh x \, dx$.

14. The region between the curve $y = \sqrt{x}, 0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

15. Determine whether the sequence $a_n = \dfrac{2n+1}{3n+1}$ is non-decreasing and if it is bounded from above.

16. Describe the level surface of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 1}$.

17. The plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at $(1, 2, 5)$.

18. Find $\dfrac{\partial z}{\partial y}$ if the equation $yz - \ln z = x + y$ defines z as a function of the two independent variables x and y and the partial derivatives exists.

19. Is the area under the curve $y = \ln x / x^2$ from $x = 1$ to $x = \infty$ finite ? If so, what is it ?

20. Evaluate $\int_0^1 \dfrac{dx}{\sqrt{3 + 4x^2}}$.

21. Write the polar form of the circle $x^2 + (y-3)^2 = 9$.

22. Draw the polar curve $r = 2 \cos \theta$.

23. Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.

24. Find the volume of $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$.

26. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

27. Does the sequence whose nth term is $\left(\frac{n+1}{n-1}\right)^n$ converge ? If so, find its limit.

28. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^{n-1} - 1}{3^n}$.

29. Find the linearization of $f(x, y, z) = x^2 - xy + 3\sin z$ at the point $(2, 1, 0)$.

30. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = 2x + 2y - z^2, x = r/s, y = r^2 + \ln s, z = 2r$.

31. Find the length of the curve $x = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 3$.

32. Show that $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converges.

33. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the length of the cardioid $r = 1 - \cos\theta$.

35. a) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

b) Show that $\tanh^2 x = 1 - \operatorname{sech}^2 x$

36. a) Using partial differentiation find $w'(0)$ if $w = xy + z, x = \cos t, y = \sin t, z = t$.

b) If $f(x-y, y-z, z-x) = 0$, show that $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$.

(2 × 10 = 20 marks)

C 4161 (Pages : 4) Name.......................................

Reg. No...................................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What is the minimum value of $f(x) = \cos x$, on $[-\pi/2, \pi/2]$.

2. Evaluate $\lim_{x \to \infty} \left(5 + \frac{1}{x}\right)$.

3. Find the average value of $f(x) = 4 - x^2$ on $[0, 3]$.

4. Evaluate $\int_1^{32} x^{-6/5}\, dx$.

5. Evaluate the sum $\sum_{k=1}^{2} \frac{6k}{k+1}$.

6. Suppose that f is integrable and that $\int_1^2 f(x)\, dx = -4$, $\int_1^5 f(x)\, dx = 6$. Evaluate $\int_2^5 f(x)\, dx$.

7. How do you define and calculate the area of the region between the graphs of two continuous functions ?

8. How do you define and calculate the length of the graph of a smooth function over a closed interval ?

9. How do you define and calculate the area of the surface swept out by revolving the graph of a smooth function $y = f(x), a \le x \le b$, about the x-axis ?

Turn over

10. What is the moment about the origin of a thin road along the x-axis with density function $\delta(x)$?

11. Define the work done by a variable force $F(x)$ directed along the x-axis from $x = a$ to $x = b$.

12. State Hooke's Law for springs.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. State the Max-Min Theorem for Continuous Functions.

14. Verify Mean Value Theorem for the function $f(x) = x^2 + 2x - 1$, in the interval $[0, 1]$.

15. Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 0$.

16. Evaluate $\int_{-4}^{4} |x| \, dx$.

17. Using substitution evaluate the integral $\int_0^3 \sqrt{y + 1} \, dy$.

18. Find the area of the region enclosed by the line $y = 2$ and curve $y = x^2 - 2$.

19. The region between the curve $y = \sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

20. Set up an integral for the length of the curve $y = x^2$, in the interval $-1 \le x \le 2$.

21. Set up an integral for the area of the surface generated by revolving the curve $y = \tan x$, $0 \le x \le \pi/4$; about x-axis.

22. Show that the center of mass of a straight, thin strip or rod of constant density lies halfway between its two ends.

23. Find the work done by a force of $F(x) = 1/x^2$ N along the x-axis from $x = 1\ m$ to $x = 10\ m$.

24. What is the Center of Mass of a thin plate covering a region in the xy-plane?

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

Each question carries 5 marks.

25. Given $f'(x) = (x-1)^2 (x+2)^2$.

    (a) What are the critical points of f?

    (b) On what intervals is f increasing or decreasing ?

26. Find the asymptotes of the curve :

    $$y = \frac{x+3}{x+2}.$$

27. State and prove Rolle's Theorem.

28. Find two positive numbers whose sum is 20 and whose product is as large as possible.

29. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x, -1 \le x \le 2$.

30. A pyramid 3 m high has a square base that is 3 m on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m ona side. Find the volume of the pyramid.

31. Find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1, \quad 0 \le x \le 1$.

32. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$.

33. Find the moment about the x-axis of a wire of constant density that lies along the curve $y = \sqrt{x}$ from $x = 0$ to $x = 2$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. (a) Sketch the Graph of $y = (x-2)^3 + 1$. Include the co-ordinates of inflection point in the graph.

(5 marks)

(b) Find the intervals on which $g(x) = -x^3 + 12x + 5,\ -3 \le x \le 3$ is increasing and decreasing. Where does the function assume extreme values and what are these values ?

(5 marks)

35. (a) If f is continuous at every point of $[a, b]$ and F is any antiderivative of f on $[a, b]$, then prove that

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

(5 marks)

(b) A surveyor, standing 30ft from the base of a building, measures the angle of elevation to the top of the building to be 75°. How accurately must the angle be measured for the percentage error in estimating the height of the building to be less than 4 % ?

(5 marks)

36. (a) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x},\ 1 \le x \le 2$, about the x-axis.

(5 marks)

(b) Find the center of mass of a thin plate of constant density δ covering the region bounded above by the parabola $y = 4 - x^2$ and below by the x-axis.

(5 marks)

[2 × 10 = 20 marks]