D 12035-A (Pages : 4) Name.......................................

Reg. No...................................

THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MEC 3C 03—MATHEMATICAL ECONOMICS

(2019—2020 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MEC 3C 03—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. When $Y''(t) = 10$, $Y(t)$ will be :

(A) 1. (B) 0.

(C) $5t^2$. (D) $5t^2 + tc_1 + C$.

2. What is the degree of $\left(\frac{dy}{dx^2}\right)^6$:

(A) 2. (B) 3.

(C) 4. (D) 6.

3. Curve that is also known as equal product curve is :

(A) Indifference Curve. (B) Isoquants.

(C) Demand Curve. (D) None of the above.

4. The order of $I_t = a(y_{t-1} - y_{t-2})$ is :

(A) 1. (B) –1.

(C) 2. (D) –2.

5. When $\sigma = 0$, substitution will be :

(A) Possible. (B) Sometimes possible.

(C) Impossible. (D) Cannot say.

6. The third stage in the law of variable proportion is called :

(A) Increasing returns. (B) Diminishing returns.

(C) Negative returns. (D) Proportional return.

7. Given $\frac{\partial Q}{\partial x_1}$, where x_1 is input represents :

(A) APx_1. (B) MPx_1.

(C) MRTS. (D) MRS.

8. When the total product is maximum, marginal product will be ?

(A) Minimum. (B) Zero.

(C) Maximum. (D) Negative.

9. When demand for a good is give by Q = 40 – P, the maximum amount that would be demanded at nil price is :

(A) 1. (B) 0.

(C) 40. (D) 400.

10. Technical relationship between input and output is called :

(A) Elasticity. (B) Production function.

(C) Input function. (D) None of the above.

11. Higher isoquants represents higher :

(A) Profit. (B) Output.

(C) Cost. (D) None of the above.

12. What is the degree of $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^6 = 10 - Y$:

(A) 3. (B) 4.

(C) 2. (D) 6.

13. The producer will be in equilibrium when :

(A) $MRTS = \frac{Px}{Py}$. (B) $MRTS > \frac{Px}{Py}$.

(C) $MRTS < \frac{Px}{Py}$. (D) None of the above.

14. Harred model explains ———— growth of the economy.

(A) Static. (B) Dynamic.

(C) Equilibrium. (D) Balanced.

15. A line that connects various equilibrium points of producer is :

(A) Isocost line. (B) Isoquants.

(C) Expansion path. (D) Price line.

16. Cobb Douglas production function is of degree :

(A) One. (B) Two.

(C) Three. (D) Four.

17. Iso quants are downward sloping and ———— to the origin.

(A) Convex. (B) Concave.

(C) Vertical. (D) Horizontal.

18. What is the order of $\frac{d^3y}{dx^3} + \left(x^2 Y\right)\frac{d^2y}{dx^2} - 4Y^4 = 0$:

(A) First. (B) Second.

(C) Third. (D) Fourth.

19. MRTS is the slope of :

(A) Production function. (B) Priceline

(C) Isocostline. (D) Isoquant.

20. For producer which is rational stage for producer in law of variable proportion :

(A) First. (B) Second.

(C) Third. (D) None of the above.

D 12035 (Pages : 2) Name.......................................

Reg. No...................................

THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MEC 3C 03—MATHEMATICAL ECONOMICS

(2019—2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Define difference equation with an example.
2. Write down the standard form of a first order linear differential equation. What is the general equation used to solve a first order linear difference equation ?
3. Distinguish between order and degree of differential equations with example.
4. What is Marginal Rate of Technical Substitution ?
5. Write the first order partial derivatives for the function $q = 5\,K^{0.4}L^{0.6}$.
6. What do you mean by elasticity of substitution ?
7. Explain the properties of Cobb-Douglas Production Function.
8. What are returns to scale ?
9. What is Certainty-Equivalent approach ?
10. Explain Law of Variable Proportions.
11. What do you mean by a Decision Tree ?
12. State briefly the properties of isoquants.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Solve the differential equation $\frac{dy}{dx} + \frac{1}{x}y = x$.
14. Solve the difference equation $3y_{x+1} = 3y_{x-7}$.

Turn over

15. Explain Economic Regions of Production Using Ridge Lines.

16. How will you optimize Cobb-Douglas production Function ?

17. Discuss the Probability Distribution approach.

18. What do you mean by solution of a difference equation ? Distinguish between general solution and particular solution of differential equations.

19. Explain producer's Equilibrium with graphical representation.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries **11** *marks.*

20. What do you mean by Investment Appraisal ? Explain Various Investment Appraisal methods.

21. Discuss how differential equations are used in the area of Economics.

(1 × 11 = 11 marks)

D 12034 (Pages : 3) Name.......................................

Reg. No...................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 3C 03—MATHEMATICS – 3

(2019–2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Evaluate $\int_0^1 \left(t\hat{i} + 3t^2\hat{j} + 4t^3\hat{k}\right)dt.$

2. The position of a moving particle is $\bar{r}(t) = t^2\hat{i} + t\hat{j} + t^3\hat{k}$. Find velocity and acceleration of the particle at $t = 2$.

3. If $z = e^{-y}\cos x$ find $\frac{\partial^2 z}{\partial x\,\partial y}$.

4. Find the level surface of $F(x,y,z) = x^2 + y^2 + z^2$ passing through (1, 1, 1).

5. Evaluate $\oint_C x\,dx$, where C is the circle $x = \cos t, y = \sin t, 0 \le t \le 2\pi$.

6. Show that curl $\bar{r} = \vec{0}$.

7. State Green's theorem in the plane.

8. Evaluate $\int_0^3\int_0^2\int_0^1 xyz\,dx\,dy\,dz.$

9. Write the equation of the circle with centre (1, 2) and radius 4 in the complex plane.

Turn over

10. Find the value of i^{2i}.

11. Evaluate $\oint_C \frac{Ze^z}{(z-3)} dz$, where C is $|z| = 2$.

12. Evaluate $\oint_C \frac{dz}{z}$, where C is $|z| = 1$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Use chain rule to find $\frac{dw}{dx}$ at (0,1, 2) for $w = xy + yz$; $x = \cos x, y \sin x, z = e^x$.

14. Find the directional derivative of $f(x,y) = \sqrt{x^2 y + 2y^2 z}$ at $(-2, 2, 1)$ in the direction of the negative z-axis.

15. Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$ using double integrals.

16. Use polar coordinates to evaluate $\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5 + x^2 + y^2} dy\, dx$.

17. Show that $f(z) = \left(2x^2 + y\right) + i\left(y^2 - x\right)$ is not analytic at any point.

18. Evaluate $\oint_C \frac{5z+7}{z^2 + 2z - 3} dz$, where C is the circle $|z - 2| = 2$.

19. Evaluate $\int \text{Re}\, z\, dz$ along a line segment from $z = 0$ to $z = 1 + 2i$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Let $\vec{F}(x, y, z) = z\hat{j} + z\hat{k}$ represents the flow of a liquid. Find the flux of $\vec{F}$ through the surface S given by that portion of the plane $z = 6 - 3x - 2y$ in the first octant oriented upward.

21. Use triple integrals to find the volume of the solid with in the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

(1 × 11 = 11 marks)

D 12033-A (Pages : 4) Name.....................................

Reg. No..................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(2019–2020 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(Multiple Choice Questions for SDE Candidates)

1. $\frac{d}{dn}(\ln kx) =$ ______________.

   (A) kx.
   (B) $\frac{1}{kx}$.
   (C) $\frac{1}{x}$.
   (D) $\frac{k}{\ln x}$.

2. $\sinh x = \frac{-3}{4}$. Then $\cosh x$ is ______________.

   (A) $\frac{1}{4}$.
   (B) $\frac{5}{4}$.
   (C) $\frac{4}{5}$.
   (D) $\frac{-3}{5}$.

3. $\frac{d}{dx}\cosh^{-1}\left(x^2\right) =$ ______________.

   (A) $2x\sinh^{-1}\left(x^2\right)$.
   (B) $\frac{2x}{\sqrt{x^2-1}}$.
   (C) $\frac{x}{x^4-1}$.
   (D) $\frac{2x}{\sqrt{x^4-1}}$.

4. $\lim_{x \to 0} \frac{a^x - b^x}{x} =$ ______________.

   (A) $\ln\left(\frac{a}{b}\right)$.
   (B) $\ln\left(\frac{b}{a}\right)$.
   (C) $\ln(ab)$.
   (D) ∞.

5. $\lim_{x \to 0} x \cot x =$ ______________.

   (A) 0.
   (B) 1.
   (C) ∞.
   (D) -1.

6. $a_1 = 2, a_{n+1} = a_n + 3$. Then $a_2 =$ ______________.

   (A) 5.
   (B) 3.
   (C) 6.
   (D) 7.

7. $a_n = \dfrac{2n+1}{3n+5}$. The $\lim\limits_{n \to \infty} a_n =$ ________________.

(A) $\frac{1}{5}$.

(B) $\frac{2}{5}$.

(C) $\frac{2}{3}$.

(D) $\frac{3}{2}$.

8. $\lim\limits_{n \to \infty} \dfrac{3^n}{n^2} =$ ________________.

(A) 3.

(B) $\frac{3}{2}$.

(C) 1.

(D) ∞.

9. If $|r| > 1$, $\lim\limits_{n \to \infty} r^n =$ ________________.

(A) 0.

(B) 1.

(C) ∞.

(D) None of these.

10. Let $\{a_n\}$ of $\{b_n\}$ is such that $a_n \leq b_n$. Then :

(A) $\sum a_n$ converges if $\sum b_n$ converges.

(B) $\sum b_n$ converges if $\sum a_n$ converges.

(C) $\sum a_n$ converges if $\sum b_n$ diverges.

(D) $\sum b_n$ diverges if $\sum a_n$ diverges.

11. $\sum\limits_{n=1}^{\infty} \dfrac{n}{n^3+1}$ is :

(A) Converges to 1.

(B) Converges to 0.

(C) Diverges.

(D) None of these.

12. If a series converges absolutely for all x. Then its radius of converges if :

(A) Finite.

(B) Infinite.

(C) Cannot be determined.

(D) None of these.

13. The series $\sum\limits_{n=0}^{\infty} x^n$ is :

(A) Converges absolutely for $|x| < 1$.

(B) Converges for $|x| > 1$.

(C) Has radius of converges $\frac{1}{2}$.

(D) None of these.

Turn over

14. Find $\frac{d^2y}{dx^2}$ if $x = \cos t, y = \sin t$:

(A) $\text{cosec}^2 t$.
(B) $-\text{cosec}^2 t$.
(C) $\text{cosec}^3 t$.
(D) $-\text{cosec}^3 t$.

15. The equation $r^2 = \sin 2\theta$ is symmetric about :

(A) x-axis.
(B) y-axis.
(C) Origin.
(D) The line $\theta = \frac{\pi}{4}$.

16. Polar equation of the circle with centre at $\left(-2, \frac{\pi}{2}\right)$ and passing through origin is :

(A) $r = -4 \sin \theta$.
(B) $r = 4 \sin \theta$.
(C) $r = 4 \cos \theta$.
(D) $r = -4 \cos \theta$.

17. $\ln\left(e^x\right) =$ ————————.

(A) e^x.
(B) x.
(C) $\frac{1}{e^x}$.
(D) $\frac{1}{x}$.

18. $\Sigma (-1)^n \frac{1}{n}$ is ————————.

(A) Converges.
(B) Diverges.
(C) Absolutely convergent.
(D) None.

19. $y = \text{sech}^2 x + \tanh^2 x$ then $\frac{dy}{dx} =$ ————————.

(A) $2 \text{ sech } x \tanh x$.
(B) 0.
(C) 1.
(D) $\text{sech}^2 x \tanh x$.

20. The series $\sum n^m x^n$ is converged if ————————.

(A) $x > 1$ and $x = 1$ when $m < -1$.
(B) $x > 1$ and $x = 1$ when $m > -1$.
(C) $x < 1$ and $x = 1$ when $m < -1$.
(D) $x < 1$ and $x = 1$ when $m > -1$.

D 12033 (Pages : 3) Name.......................................

Reg. No...................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(2019–2020 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 30.

1. Determine whether the function $f(x) = x^3 - 3x + 1$ has an inverse.

2. Find the derivatives of (a) $3^{\sqrt{x}}$; (b) $\cos^{-1}(3x)$.

3. Find the derivative of $\log\left[\frac{x^2\left(2x^2+1\right)^3}{\sqrt{5-x^2}}\right]$ when $x = 1$.

4. Evaluate $\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$.

5. Find $\lim_{x \to \infty} \frac{\log n}{n}$.

6. Determine whether the series converges. If it converges find the sum $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)$.

7. Use integral test to determine whether $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges or diverges.

8. Show that the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges.

Turn over

9. Find the Maclaurin's series of $f(x) = \cos x$.

10. Find the radius of convergence and interval of convergence of the power series $\sum_{n=0}^{\infty} n!x^n$.

11. Describe the curve represented by $x = 4\cos\theta$ and $y = 3\sin\theta, 0 \le \theta \le 2\pi$.

12. Find the angle between the two planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.

13. Find an equation in rectangular co-ordinates for the surface with the cylindrical co-ordinates $r^3 \cos 2\theta - z^2 = 4$.

14. Find a vector function that describes the curve of intersection of the cylinder $x^2 + y^2 = 4$ and the plane $x + y + 2z = 4$.

15. Evaluate $\int_0^1 r(t)\, dt$ if $r(t) = t^2 i + \frac{1}{t+1} j + e^{-t} k$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Use logarithmic differentiation to find the derivative of $y = \sqrt[3]{\frac{x-1}{x^2+1}}$.

17. Find the derivative of $y = x^2 \operatorname{sech}^{-1}(3x)$.

18. Evaluate $\int_0^1 \log x\, dx$.

19. Show that the series $\sum_{n=1}^{\infty}\left[\frac{2}{n(n+1)} - \frac{4}{3^n}\right]$ is convergent and find its sum.

20. Find the tangent lines of $r = \cos 2\theta$ at the origin.

21. Find the length of the Cardioid $r = 1 + \cos\theta$.

22. Find the parametric equations for the line of intersection of the planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.

23. Find the velocity vector, acceleration vector and speed of a particle with position vector :

$r(t) = \sqrt{t}\, i + tt^2 j + e^{2t} k, t \geq 0.$

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Find the derivative of $\sec^{-1}\left(e^{-2x}\right)$.

(b) Evaluate $\lim\limits_{x \to 0^{+}} \left(\frac{1}{x}\right)^{\sin x}$

25. (a) Find the area S of the surface obtained by revolving the circle $r = \cos\theta$ about the line $\theta = \pi/2$.

(b) Show that the surface area of a sphere of radius r is $4\pi r^2$.

26. (a) Determine whether the series $\sum\limits_{n=1}^{\infty} \frac{n!}{n^n}$ is convergent or divergent.

(b) Show that sequence $\left\{\frac{2^n}{n!}\right\}$ is convergent and find its limit.

27. (a) Find an equation in rectangular co-ordinates for the surface with spherical equation $\rho = 4\cos\phi$.

(b) A moving object has an initial position and an initial velocity given by the vectors $r(0) = i + 2j + k$ and $v(0) = i + 2k$. Its acceleration at time t is $a(t) = 6t\, i + j + 2k$. Find its velocity and position at time t.

(2 × 10 = 20 marks)

D 11861-A (Pages : 4) Name.......................................

Reg. No...................................

THIRD SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

ME 3C 03—MATHEMATICAL ECONOMICS

(2014–2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

ME 3C 03—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. In the production function $Q = AK^{\alpha}L^{\beta}$, the variable L denotes :

(A) Leisure. (B) Less.

(C) Labour. (D) Loss.

2. Curve that is also known as equal product curve is :

(A) Indifference curve. (B) Isoquants.

(C) Demand Curve. (D) None of the above.

3. A homogeneous production function with degree one corresponds :

(A) Constant returns. (B) Diminishing returns.

(C) Increasing returns. (D) Negative returns.

4. Given $\frac{\partial Q}{\partial x_1}$, where x_1 is input represents :

(A) APx_1. (B) MPx_1.

(C) MRTS. (D) MRS.

5. When the total product is maximum, marginal product will be ?

(A) Minimum. (B) Zero.

(C) Maximum. (D) Negative.

6. When demand for a good is give by $Q = 40 - P$, the maximum amount that would be demanded at nil price is :

(A) 1. (B) 0.

(C) 40. (D) 400.

7. Combinations of two inputs resulting in equal total output is :

(A) Isoquant. (B) Isocost.

(C) Indifference curve. (D) Priceline.

8. Which of the following shows constant returns to scale ?

(A) Cobb Douglas production function. (B) CES production function.

(C) Both (A) & (B). (D) None of the above.

9. $\frac{Q}{x_2}$, where x_2 denotes inputs corresponds to :

(A) MPx_2. (B) MPx_1.

(C) APx_2. (D) APx_1.

10. IN the CES production function $Q\left[\delta C^{-\alpha}+(1-\delta)N^{-\infty}\right]\frac{-\upsilon}{\infty}$ the term C denotes :

(A) Constant. (B) Output.

(C) Capital. (D) Population.

11. Which of the following is called product exhaustion theorem ?

(A) Eulers's theorem. (B) Cobb-Douglas function.

(C) CES. (D) Translog.

12. Demand function $Q=f(P)$ if point elasticity $\in=-1$ for all $P>0$ will be :

(A) CP. (B) $\frac{c}{p}$.

(C) P. (D) None of the above.

13. What is the order of $\left(\frac{d^2y}{dt^2}\right)^7+\left(\frac{d^3y}{dt^3}\right)^5=100y$:

(A) 2. (B) 3.

(C) 5. (D) 7.

14. A line for linear equation should begin from :

(A) Origin. (B) X-axis.

(C) Y-axis. (D) Any of the above.

15. Euler's theorem is valid only if factors are paid reward on the basis of value of :

(A) Average product.
(B) Total Product.
(C) Marginal product.
(D) None of the above.

16. The order of $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=25x$ is :

(A) First.
(B) Second.
(C) Third.
(D) None of the above.

17. The value of Rs. 100 at 10% interest for two years :

(A) 110.
(B) 111.
(C) 121.
(D) 130.

18. Warranted growth rate in the Harved model is :

(A) SYt.
(B) $\frac{s}{a-s}$.
(C) $\frac{a}{a-s}$.
(D) $\frac{a-s}{a}$.

19. Functional relationship between input and output is called :

(A) Isoquants.
(B) Isocost.
(C) Input function.
(D) Production function.

20. What is the order of $\frac{d^3Y}{dx^3}+\left(x^2Y\right)\frac{d^2Y}{dx^2}-4Y^4=0$:

(A) First.
(B) Second.
(C) Third.
(D) Fourth.

D 11861 **(Pages : 3)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

ME 3C 03—MATHEMATICAL ECONOMICS

(2014–2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Write the order and degree of the differential equation $\left(\frac{d^2y}{dt^2}\right)^5 + \left(\frac{d^3y}{dt^3}\right)^4 - 36y = 0.$

2. Solve $\frac{dy}{dt} = y + 1.$

3. Write the general solution of the difference equation $y_t + \frac{1}{4}y_{t-1} = 0$.

4. Mathematically, how will you represent a production function.

5. Define the term Marginal product.

6. Define Marginal rate of technical substitution.

7. State Euler's theorem.

8. Check the returns to scale of the production function $Q = A\,K^{0.3}\,L^{0.7}$.

9. For the production function $P = AL^{\alpha}K^{\beta}$, prove that labour L and capital K are essential factors of production.

10. Define the term Pay-back period.

11. Write any two advantages of ARR method.

12. Write a short note on Decision tree approach.

(12 × 1 = 12 marks)

Turn over

Part B

Answer any **six** *questions.*
Each question carries 3 *marks.*

13. Solve the differential equation $\frac{dy}{dt} = \frac{y^2}{t^2}$.

14. Find the demand function $Q = f(p)$ if point elasticity is $\frac{-(5p + 2p^2)}{Q}$ and $Q = 500$ when $P = 10$.

15. Solve the difference equation $y_t = y_{t-1} - 25$ given $y_0 = 40$.

16. Explain why an Isoquant slopes downward from left to right.

17. Define the term Producers equilibrium.

18. Write the general form of CES production function.

19. Given the production function $Q = AK^{\alpha}L^{\beta}$, find out Marginal productivity of capital (K) and labour (L).

20. A project of Rs. 20 lakhs yielded annually a profit of Rs. 4 lakhs after depreciation at 10% and is subject to income tax at 40%. Calculate pay-back period of this project.

21. Write any three advantages of NPV method.

(6 × 3 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 *marks.*

22. Show that $(12y + 7t + 6)\,dy + (7y + 4t - 9)\,dt = 0$ is exact and hence solve.

23. For the following data find the level of income y_t for any period and warranted rate of growth :

    $I_t = 4.2(y_t - y_{t-1})$; $s_t = 0.2\,y_t$; $y_0 = 5600$.

24. Show that the marginal product is always equal to the average product when average product is maximum.

25. If $P = f(A, B)$ is a linear homogeneous production function, the Euler's theorem $P = A\frac{\partial P}{\partial A} + B\frac{\partial P}{\partial B}$.

26. If the profit rate (price of capital) remains unaltered the ratio of the amount of capital employed per unit of labour shifts from 10 : 10 to 12 : 11, given that the rise in wages is 25%. Determine the elasticity of substitution.

27. Prove that the expression path of the Cobb-Douglas production function is linear and passes through the origin.

28. Prove that for the C.E.S. production function the marginal product is greater than zero.

29. Compute the comparative profitability of the two proposals using the method of ARR :

Particulars		Investment Proposal for	
		Machine A	Machine B
1	Initial cost (Rs.)	2,24,000	60,000
2	Estimated life (years)	5.5	8
3	Estimated sails (Rs.)	1,50,000	1,50,000
4	Costs (Rs.)		
5	Material	50,000	50,000
6	Labour	1,20,000	60,000
7	Overheads	24,000	20,000

30. Write a note on simulation approach.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*

31. For the data given below, determine (i) The market price P_t in any time period ; (ii) The equilibrium price, P_e ; (iii) The stability of time path :

$Q_{dt} = 160 - 0.8\, P_t$; $Q_{st} = -20 + 0.4\, P_{t-1}$; $P_0 = 153$.

32. Find the elasticity of substitution for the production function $P = AL^{\alpha}K^{\beta}$ with $\alpha + \beta = 1$.

33. Derive producer's equilibrium condition if the production function is $P = f(A, B)$ and the prices of the factors A and B are P_A and P_B respectively.

(2 × 10 = 20 marks)

D 11860 (Pages : 4) Name.................................

Reg. No...............................

THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MAT 3C 03—MATHEMATICS

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define ordinary differential equation.
2. The solution of the differential equation $y' - 1 + y^2$ is ———.
3. The degree of the differential equation $\frac{dy}{dx} = -2\sin x\cos x$ is ———.
4. The rank of the matrix $A = \begin{bmatrix} 0 & 2 \\ 0 & 5 \end{bmatrix}$ is ———.
5. State True *or* False : The following two matrices are equivalent :

$$\begin{bmatrix} 5 & 5 & -5 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 3 & -3 & 3 & -7 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 3 & -3 & 3 & -7 \end{bmatrix}.$$

6. The characteristic matrix of the matrix $\begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix}$ is ———.
7. Find the resultant of the vectors $\mathbf{p} = [4, -2, -3]$, $\mathbf{q} = [8, 8, 1]$, and $\mathbf{u} = [-12, -6, 2]$.
8. For the vectors $\mathbf{a} = [1, 3, 2]$, $\mathbf{b} = [2, 0, -5]$, and $\mathbf{c} = [4, -2, 1]$, find $(\mathbf{a} \cdot \mathbf{b})\,\mathbf{c}$.

Turn over

9. The vector function $\mathbf{r}(t) = (4 + t)\,\mathbf{i} + (2 + t)\,\mathbf{j}$ represents ————.

10. Let $\bar{r}(t) = 5t^2\,\bar{k}$ be the position vector of a moving particle, where $t \geq 0$ is time. Then the acceleration vector of the moving particle is ————.

11. If $\bar{v} = e^x(\cos y\mathbf{i} + \sin y\mathbf{j})$ then div $\bar{v}$ = ————.

12. When we say that a vector valued function is conservative ?

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Verify that $y = e^{4x}$ is a solution of the differential equation $\frac{dy}{dx} = 4y$.

14. Solve the initial value problem $2xy' - 3y = 0$; $y(1) = 4$.

15. Show that the equation :

$$ydx + xdy = 0$$

is exact and solve it.

16. Reduce the matrix $\begin{bmatrix} 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ to its normal form.

17. Solve completely the system of equations :

$$x + 3y - 2z = 0$$

$$2x - y + 4z = 0$$

$$x - 11y + 14z = 0.$$

18. Find the eigen values of :

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}.$$

19. Prove that $(\mathbf{u} + \mathbf{v})' = \mathbf{u}' + \mathbf{v}'$

20. Find $\frac{df}{ds}$ in the direction of the vector $4i + 4j - 2k$ at the point $(1, 1, 2)$ if $f(x, y, z) = x^2 + y^2 - z$.

21. Show that $\mathbf{F}(x, y) = (\cos y + y \cos x)\,\mathbf{i} + (\sin x - x \sin y)\,\mathbf{j}$ is a conservative vector field.

22. Find the unit tangent vector at a point t to the curve $\mathbf{r} = a \cos t\,\mathbf{i} + a \sin t\,\mathbf{j}$.

23. Find unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

24. Verify that $w = x^2 - y^2$ satisfies Laplace's equation $\nabla^2 w = 0$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 marks.

25. Write in the linear form and then solve $\sin 2x \dfrac{dy}{dx} = y + \tan x$.

26. Determine the rank of the following matrix, by reducing to echelon form :

$$\begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}.$$

27. Show that the system of equations :

$$\begin{aligned} x + 2y + z &= 2 \\ 3x + y - 2z &= 1 \\ 4x - 3y - z &= 3 \\ 2x + 4y + 2z &= 4 \end{aligned}$$

is consistent and hence solve them.

28. Find the eigen values and the eigen vector corresponding to the largest eigen value of the matrix

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}.$$

29. Using Cayley-Hamilton theorem find the inverse of $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$.

30. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ with

$$\mathbf{F}(r) = 5z\mathbf{i} + xy\mathbf{j} + x^2z\mathbf{k}$$

along the straight - line segment C : $t\mathbf{i} + t\mathbf{j} + t\mathbf{k},\ 0 \le t \le 1$.

31. Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ by the divergence theorem where $\mathbf{F} = [x^2, 0, z^2]$ and S is the surface of the box given by the inequalities $|x| \le 1, |y| \le 3, |z| \le 2$.

32. Evaluate the flux integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ where $\mathbf{F} = \left[e^{2y}, e^{-2z}, e^{2x}\right]$, and

$S : \mathbf{r} = [3 \cos u, 3 \sin u, v],\ 0 \le u \le \frac{1}{2}\pi,\ 0 \le v \le 2$.

33. Apply Green's theorem to evaluate $\oint_C (2x^2 - y^2)\, dx + (x^2 + y^2)\, dy$, where C is the boundary of the area enclosed by the x-axis and the upper half off the circle $x^2 + y^2 = a^2$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the orthogonal trajectories of the family of curves $x^2 + y^2 = c^2$.

35. Investigate for what values of a, b the system of equations :

$$x + y + 2z = 2$$
$$2x - y + 3z = 10$$
$$5x - y + az = b$$

have unique solution.

36. Calculate the line integral $\oint_C \mathbf{F}.\mathbf{r}'(s)\, ds$ using Stoke's theorem, where $\mathbf{F} = [-5y, 4x, z]$, and C is the circle $x^2 + y^2 = 4, z = 1$.

(2 × 10 = 20 marks)

D 11859-A (Pages : 4) Name.......................................

Reg. No...................................

THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2021

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

(2014—2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

(Multiple Choice Questions for SDE Candidates)

1. $\int \tan x \, dx =$ ____________.

(A) $\ln|\cos x| + c$.

(B) $\ln|\sec x| + c$.

(C) $\sec^2 x + c$.

(D) $\ln|\sin x| + c$.

2. $\int a^x \, dx =$ ____________.

(A) $\dfrac{a^x}{\ln a}$.

(B) $a^x \ln a$.

(C) $\dfrac{\ln a}{a^x}$.

(D) $\dfrac{a^{x+1}}{x+1}$.

3. $\coth^{-1} x =$ ____________.

(A) $\dfrac{1}{\tanh x}$.

(B) $\tanh^{-1}\left(\dfrac{1}{x}\right)$.

(C) $\coth^{-1}\left(\dfrac{1}{x}\right)$

(D) $\dfrac{1}{\tanh^{-1}\left(\dfrac{1}{x}\right)}$.

4. $\lim\limits_{x \to 0} \dfrac{\sin x}{x^2} =$ ____________.

(A) -1.

(B) 0.

(C) ∞.

(D) $-\infty$.

5. Range of $\tanh x$ is ____________.

(A) $[-1, 1]$.

(B) $(-1, 1)$.

(C) $(-\infty, \infty)$.

(D) $(0, \infty)$.

6. $\lim\limits_{n \to \infty} \dfrac{2n}{n+1} =$ ____________.

(A) 0.

(B) $\dfrac{2}{3}$.

(C) 2.

(D) ∞.

7. $\lim_{n \to \infty} x^{1/n} (x > 0) =$ ____________.

(A) 0.

(B) 1.

(C) ∞.

(D) None of these.

8. Fourth term of the sequence $\left\{ \frac{(-1)^{n+1}}{2^{n+1}} \right\}$ is :

(A) $\frac{1}{2^5}$.

(B) $\frac{-1}{2^5}$.

(C) $\frac{1}{2^4}$.

(D) $\frac{-1}{2^4}$.

9. The radius of convergence of the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ is ____________.

(A) 0.

(B) 1.

(C) 2.

(D) ∞.

10. Co-efficient of x^2 in the Maclaurin's series expansion of $f(x) = l_n (x+1)$ is :

(A) -1.

(B) $-\frac{1}{2}$.

(C) $\frac{1}{2}$.

(D) 1.

11. Equation of the directrix of the parabola $y^2 = 8x$ is :

(A) $x = 2$.

(B) $y = 2$.

(C) $x = -2$.

(D) $y = -2$.

12. $x^2 + 2xy + y^2 + 2x - y + 2 = 0$ represent :

(A) Parabola.

(B) Ellipse.

(C) Hyperbola.

(D) Circle.

13. Find $\frac{d^2y}{dx^2}$ if $x = \cos t, y = \sin t$:

(A) $\text{cosec}^2\, t$.

(B) $-\text{cosec}^2\, t$.

(C) $\text{cosec}^3\, t$.

(D) $-\text{cosec}^3\, t$.

14. The equation $r^2 = \sin 2\theta$ is symmetric about :

(A) x-axis. (B) y-axis.

(C) Origin. (D) The line $\theta = \frac{\pi}{4}$.

15. $\Sigma(-1)^n \frac{1}{n}$ is ————————.

(A) Converges. (B) Diverges.

(C) Absolutely convergent. (D) None.

16. $\lim_{x \to \infty} \left(\frac{1}{n^2}\right)^{\frac{1}{n}} =$ ————————.

(A) 0. (B) 1.

(C) ∞. (D) None.

17. If $f(x) = 5x - 4, \quad \text{if } x < 2$ $\quad \lim_{x \to 2} f(x) =$ ————————.
$\qquad 2(x^2 - 1), \quad \text{if } x > 2$

(A) 0. (B) 6.

(C) 14. (D) Does not exist.

18. $y = \sin(\sin x)$. Then $\frac{dy}{dx} - \cos(\sin x)\cos x =$ ————————.

(A) 0. (B) $\cos x$.

(C) $\sin x$. (D) 1.

19. The function $f(x) = |x|$ is ————————.

(A) Continuous at $x = 0$. (B) Discontinuous at $x = 0$.

(C) Differentiable at $x = 0$. (D) Not differentiable at $x = 0$.

20. The series $\sum n^m x^n$ is converged if ————————.

(A) $x > 1$ and $x = 1$ when $m < -1$. (B) $x > 1$ and $x = 1$ when $m > -1$.

(C) $x < 1$ and $x = 1$ when $m < -1$. (D) $x < 1$ and $x = 1$ when $m > -1$.

D 11859 **(Pages : 4)** **Name....................................**

Reg. No..................................

THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2021

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type Questions)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.

2. Find $\lim_{n \to \infty} \frac{4 - 7n^6}{n^6 + 3}$.

3. The inverse function of $\ln(x), x > 0$ is ....

4. If $f(x) = x^3 - 2$, find the value of $\frac{df^{-1}}{dx}$ at $x = f(2) = 6$.

5. Find a formula for the n^{th} term of the sequence $1, -4, 9, -16, 25,$ ....

6. The centre-to-focus distance in the case of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is ....

7. Write a parametrization of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

8. $\lim_{n \to \infty} \frac{\ln n}{n} =$ ....

Turn over

9. Give an example of a conditionally convergence sequence.

10. Define a harmonic series.

11. If $\log_a x = \frac{\ln(x)}{k}$, then $k =$

12. The polar equation of the circle $x^2 + y^2 = 25$ is ....

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find the focus of the parabola $x^2 = 100\,y$.

14. Find $\lim\limits_{x \to 0^+} x \cot x$.

15. Find $\lim\limits_{n \to \infty} \frac{\cos n}{n}$.

16. State non-decreasing sequence theorem.

17. For what values of x do the power series $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ converge ?

18. Evaluate $\lim\limits_{x \to 0} \frac{x - \sin x}{x^2}$.

19. Determine the conic section from the equation $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$.

20. Identify the geometric figure in the cartesian plane represented by the polar equation $r\cos(\theta - \pi/3) = 2$.

21. Test the convergence of $\sum\limits_{n=1}^{\infty} \frac{n^2}{2^n}$.

22. For what values of x do $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$ converge ?

23. Write the formula for finding the area using polar co-ordinates.

24. Find the polar equation of the hyperbola with eccentricity $3/2$ and directrix at $x = 2$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Using the deflation of natural logarithm, prove that $\ln(xy) = \ln x + \ln y$.

26. Evaluate $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

27. Find a formula for the n^{th} partial sum of the series $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + ... + \frac{2}{3^{n-1}}$ and use it to find the serie's sum if the series converges.

28. Use integral test to show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$.

29. Show that the point $(2, 3\pi/4)$ lies on the curve $r = 2 \sin 2\theta$.

30. Find the Maclaurin series for the function $f(x) = \sinh x$.

31. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$.

32. Calculate the value of e with an error less than 10^{-6}.

33. Draw the curve represented by the parametric equations $x = \cos t, y = -\sin t, t \in [0, \pi]$.

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the area of the surface generated by revolving the right handed loop of the lemniscate $r^2 = \cos 2\theta$ about y-axis.

35. Find the Taylor series generated by $f(x) = 1/x$ at $a = 2$. Where if anywhere, does the series converges to $1/x$?

36. a) The x and y axes are rotated through an angle of $\pi/4$ radians about the origin. Find the converted equation of the hyperbola $2xy = 9$ in the new co-ordinates.

b) Evaluate $\lim\limits_{x \to \left(\frac{\pi}{2}\right)^{-}} \frac{\sec x}{1 + \tan x}$.

(2 × 10 = 20 marks)

D 73758 (Pages : 4) Name.................................

Reg. No.................................

THIRD SEMESTER (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION NOVEMBER 2019

Mathematics

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all** *the* **twelve** *questions.*
Each question carries 1 *mark.*

1. Define differential equation.

2. The solution of the differential equation $y' = ky$ is ———.

3. The order of the differential equation $\frac{dy}{dx} = \sin x$ is ———.

4. The rank of the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ is ———.

5. State True *or* False : The following two matrices are equivalent :

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 3 & -3 & 3 & -7 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 3 & -3 & 3 & -7 \end{bmatrix}.$$

6. The characteristic matrix of the matrix $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ is ———.

7. Let $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $b = 3\mathbf{j}$, and $\mathbf{c} = 4\mathbf{i} + \mathbf{j} - \mathbf{k}$. Then $(\mathbf{a} + \mathbf{b}) + \mathbf{c} =$ ———.

8. For the vectors $\mathbf{a} = [1, 3, 2]$, $\mathbf{b} = [2, 0, -5]$ and $\mathbf{c} = [4, -2, 1]$, find $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$.

9. The vector function $\mathbf{r}(t) = 3\cos t\,\mathbf{i} - 4\sin t\,\mathbf{j}\ (0 \le t \le 2\pi)$ represents ————.

10. Let $\mathbf{r}(t) = 4t^2\,\mathbf{k}$ be the position vector of a moving particle, where $t \ge 0$ is time. The the acceleration vectort of the moving particle is ————.

11. If $\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ then div $\mathbf{v}$ = ————.

12. When we say that a vector valued function is irrotational ?

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Verify that $y = e^{3x}$ is a solution of the differential equation $\frac{dy}{dx} = 3y$.

14. Solve the initial value problem $\frac{dy}{dx} - y\tan 2x = 0 : y(0) = 2$.

15. Show that the equation :

$$(1 + 4xy + 2y^2)\,dx + (1 + 4xy + 2x^2)\,dy = 0$$

is exact and solve it.

16. Reduce the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ to its normal form.

17. Solve completely the system of equations :

$$x + 3y - 2z = 0$$

$$2x - y + 4z = 0$$

$$x - 11y + 14z = 0$$

18. Find the eigen values of :

$$\begin{matrix} 5 & 1 & 1 \\ 1 & 3 & 1 \end{matrix}$$

19. Prove that $(\mathbf{u} \times \mathbf{v})' = \mathbf{u}' \times \mathbf{v} + \mathbf{u} \times \mathbf{v}'$.

20. What is the maximum possible $\frac{df}{ds}$ at the point (1, 4, 2), if $f(x, y, z) = x^2 + y^2 - z$.

21. Show that $\mathbf{F} = (2xy + z^3)\,\mathbf{i} + x^2\,\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative vector field.

22. Find the unit tangent vector to the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at the point $t = 2$.

23. Find unit normal to the surface $x^2y + 2xz = 4$ at the point (2, – 2, 3).

24. Verify that $w = x^2 - y^2$ satisfies Laplace's equation $\nabla^2 w = 0$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Solve the linear differential equation $y' - y = e^{2x}$.

26. Determine the rank of the following matrix, by reducing to echelon form

$$\begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}.$$

27. Test the following system of equations for consistency and solve it, if it is consistent.

$$x + y + z = 6$$
$$x + y + 2z = 5$$
$$3x + y + z = 8.$$

28. Find the eigen values and the eigen vector corresponding to the largest eigen value of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & -1 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

29. Using Cayley-Hamilton theorem, find the inverse of :

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

30. If $\mathbf{F} = (3x^2 + 6y)\,\mathbf{i} - 14\,yz\,\mathbf{j} + 20\,xz^2\,\mathbf{k}$, evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the curve from (0, 0, 0) to (1, 1, 1) with parametric form $x = t,\ y = t^2,\ z = t^3$.

31. Evaluate the surface integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dA$ by the divergence theorem where $\mathbf{F} = [x^2, 0, z^2]$ and S is the surface of the box given by the inequalities $|x| \le 1, |y| \le 3, |z| \le 2$.

32. Evaluate the flux integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dA$ where $\mathbf{F} = [3x^2, y^2, 0]$, and

$S : \mathbf{r} = [u, v, 2u + 3v], 0 \le u \le 2, -1 \le u \le 1.$

33. Verify Green's theorem in the plane for $\oint_C (xy\,dx + x^2 dy)$, where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.

(6 × 5 = 30 marks)

Part D

Answer any two *questions.*
Each question carries 10 *marks.*

34. Solve $\dfrac{dy}{dx} + x\sin^2 y = x^3\cos^2 y$.

35. Investigate for what values of a, b the system of equations :

$$x + y + 2z = 2$$
$$2x - y + 3z = 10$$
$$5x - y + az = b$$

have no solution.

36. Verify $\iint_S (\text{curl}\,\mathbf{F})\cdot\mathbf{n}\,dA = \oint_C \mathbf{F}\cdot\mathbf{r}'(s)\,ds$ where $\mathbf{F} = [z^2, 5x, 0]$, S is the square $0 \le x \le 1,\ 0 \le y \le 1,\ z = 1$ and C is the boundary of S.

(2 × 10 = 20 marks)

D 73757 (Pages : 4) Name.......................................

Reg. No...................................

THIRD SEMESTER (CUCBCSS—UG) [SPECIAL] DEGREE EXAMINATION NOVEMBER 2019

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Write the parametric equations of the circle $x^2 + y^2 = 1$.

2. Find the Taylor series for $f(x) = e^x$ at $x = 0$.

3. Find the foci of the hyperbola $\frac{y^2}{4} - \frac{x^2}{5} = 1$.

4. Evaluate $\int_0^{\pi} \frac{\sin t}{2 - \cos t}\, dt$.

5. Examine whether $xy - y^2 - 5y + 1 = 0$ represents a parabola, ellipse or hyperbola.

6. Prove that $e^{x + \ln 2} = 2e^x$.

7. Evaluate $\frac{d}{dt}\left(\tanh \sqrt{1 + t^2}\right)$.

8. Examine whether $\sum_{n=1}^{\infty} \frac{n+1}{n}$ converges or diverges.

9. Find the Taylor polynomial of order zero generated by $f(x) = \frac{1}{x}$ at $a = 2$.

Turn over

10. Find the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

11. State Leibniz's Theorem for an alternating series.

12. Evaluate $\int_0^{\pi/2} e^{\sin x} \cos x \, dx$.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Find $\frac{dy}{dx}$ if $y = x^x, x > 0$.

14. Prove that the alternating series

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ converges.

15. Examine whether $x^2+xy+y^2-1=0$ represents a parabola, ellipse or hyperbola.

16. Show that ln x grows slower than x as $x \to \infty$.

17. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3+2\sin\theta} d\theta$.

18. Find the Taylor series for $f(x) = e^x$ at $x = 0$.

19. Graph the set of points whose polar co-ordinates satisfy the conditions $1 \le r \le 2$ and $0 \le \theta \le \pi/2$.

20. For what values of x do the series $\sum_{n=0}^{\infty} n!\,x^n$ converges.

21. Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$.

22. Define absolute convergence.

23. Examine whether $\sum_{n=1}^{\infty} (-1)^{n+1}$ converges or diverges.

24. Find y if $\ln y = 3t + 5$.

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 marks.

25. Find the directrix of the parabola $r = \frac{25}{10 + 10 \cos \theta}$.

26. Find the Maclaurin's series for $f(x) = \sin 3x$.

27. Find $\lim_{x \to \infty} x^{\frac{1}{x}}$.

28. Graph the curve $r = 1 - \cos \theta$.

29. Evaluate $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

30. Find the tangent to the right-hand hyperbola branch $x = \sec t$, $y = \tan t$, $-\frac{\pi}{2} < t < \pi/2$ at the point $\left(\sqrt{2}, 1\right)$ where $t = \frac{\pi}{4}$.

31. Using Integral test show that the p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \dots + \frac{1}{n^p} + \dots$$ converges if $p > 1$ and diverges if $p \le 1$.

32. Find the centroid of the first quadrant of the astroid $x = \cos^3 t,\ y = \sin^3 t, 0 \le t \le 2\pi$.

33. Find the length of the Cardioid $r = 1 - \cos\theta$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the length of the astroid $x = \cos^3 t,\ y = \sin^3 t, 0 \le t \le 2\pi$.

35. Show that the Maclaurin's series for $\sin x$ converges to $\sin x$ for all x.

36. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

(2 × 10 = 20 marks)