C 21547-A (Pages : 4) Name.......................................

Reg. No...................................

FOURTH SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MEC 4C 04—MATHEMATICAL ECONOMICS

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 15**	**Maximum : 15 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MEC 4C 04—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. Regression analysis is concerned with the study of the dependence of :

   (A) Explanatory variables on one or more dependent variables.

   (B) Dependent variable on one or more explanatory variables.

   (C) Both explanatory and dependent variables on other known variables.

   (D) Two known variables.

2. A statistical relationship in itself :

   (A) Can help establish causation. (B) Can help establish direction of causation.

   (C) Cannot logically imply causation. (D) Always shows correlation.

3. The dependent variable in regression analysis is assumed to be :

   (A) Non-stochastic. (B) Constant.

   (C) Stochastic. (D) Known values.

4. Firm data collected for top 10 companies classified based on profitability for 10 years is an example of :

   (A) Cross-sectional data. (B) Time series data.

   (C) Pooled data. (D) Panel data.

5. In $Y_i = \beta_1 + \beta_2 X_i X + u_i, u_i$ can take values that are :

   (A) Only positive. (B) Only negative.

   (C) Only zero. (D) Positive, negative or zero.

6. For a regression line that parses through the conditional means of Y, $E(Y|X_i)$ is :

   (A) Always a positive value. (B) Always a negative value.

   (C) Always zero. (D) Any of the above.

7. In the simple linear regression model, the regression slope :

(A) Indicates by how many percent Y increases given a one percent increase in X.

(B) When multiplied ... the explanatory variable will give you the predicted Y ?

(C) Indicates by how many units Y increases, given a one unit increase in X.

(D) Represents the elasticity of Y on X.

8. The statement that-There can be more than one SRF representing a population regression function is :

(A) Always true. (B) Always false.

(C) Sometimes true, sometimes false. (D) Nonsense statement.

9. The least square estimators are :

(A) Period estimators. (B) Point estimators.

(C) Population estimators. (D) Popular estimators.

10. Under normality assumption of u_i, the OLS estimator are :

(A) Minimum variance unbiased. (B) Consistent.

(C) $\hat{\beta}_1$ is normally distributed. (D) All the above.

11. Rejecting a true hypothesis results in this type of error :

(A) Type I error. (B) Type II error.

(C) Structural error. (D) Hypothesis error.

12. In confidence interval estimation, $\alpha = 5\%$, this means that this interval includes the true β with probability of :

(A) 5%. (B) 50%.

(C) 95%. (D) 45%.

13. Given that the test Statistics under test-of-significance approach lies in the critical region, the null hypothesis is :

(A) Not rejected. (B) Rejected.

(C) Rejection depends on α value. (D) Rejectection depends on t value.

14. A researcher estimates the unknown demand curve $Q = \alpha + \beta P + \epsilon$ and fits the regression line $Q = \alpha - bP$ using OLS technique. The following information is given to you : Sample size = 10, TSS = 80 : ESS = 44. Which of the following is correct ?

(A) RSS = 5 *b*. C. *d*.

(B) Given that a = 4 and b = 2, demand is price unitary elastic at P = 1.5.

(C) All coefficients are statistically significant at 5% level of significance.

(D) $R^2 = 0.25$.

15. The Jarque-Bera test is a :

(A) Model mis-specification test.

(B) Residual normality test.

(C) Test of unbiasedness of estimators .

(D) Test of goodness of fit for the model.

C 21547 (Pages : 2) Name.................................

Reg. No..............................

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2022

Mathematics

MEC 4C 04—MATHEMATICAL ECONOMICS

(2019 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. What are the different types of data ?
2. Distinguish between regression and correlation ?
3. Define a random or stochastic variable with an example.
4. Discuss the concept of Sample Regression Function (SRF) with specification.
5. What are the properties of R^2.
6. Distinguish between theoretical Econometrics and Applied Econometrics.
7. What is Homoscedasticity ?
8. What do you mean by multi-collinearity ?
9. What is level of significance ?
10. Briefly state the principle of least squares.
11. Explain semi-log models.
12. What is a standardized variable ?

(8 × 3 = 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Explain the difference between statistical and deterministic relationship.

14. What is Gaus-Markov Theorem ?

15. Derive R^2.

16. Explain the properties of OLS Estimators under the Normality Assumption for Ui.

17. Explain how regression is compared ANOVA.

18. Derive the relationship between R^2 and $\bar{R}^2$.

19. What are the main guidelines for the choice of a functional form of regression ?

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Explain the traditional methodology of Econometrics to proceed the analysis of an economic problem.

21. Explain the statistical properties of OLS estimators ?

(1 × 11 = 11 marks)

C 21546

(Pages : 2)

Name.......................................

Reg. No..................................

FOURTH SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 4C 04—MATHEMATICS – 4

(2019 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Write the order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+4y=\sin x$.

2. Verify that $y=xe^x$ is a solution of $y''-2y'+y=0$.

3. Show that $\left(25x^2-5y\right)dx+\left(3y^2-5x\right)dy=0$ is an exact differential equation.

4. Find the integrating factor corresponding to the differential equation $\frac{dy}{dx}+y\tan x=\cos x$.

5. Reduce $\frac{dy}{dx}=\left(y-2x^2\right)-7$ to an equation with separable variables.

6. Find the general solution of $y''-y'-2y=0$.

7. Find the particular integral of $y''+5y'+6y=e^{2x}$.

8. Find the Laplace transform of $\sin 3t\cos 2t$.

9. Find the Laplace transform of $e^{-3t}t^3$.

10. Write the inverse Laplace transform of $\frac{s}{s^2+16}$.

11. Show that the functions $f_1(x)=x^3$ and $f_2(x)=x^2+1$ are orthogonal on $[-1,1]$.

12. Show that the partial differential equation $3\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}$ is parabolic.

(8 × 3= 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Solve $(1+x)y\,dx+(1-y)x\,dy=0$.

14. Solve $\left(x^2+y^2\right)\frac{dy}{dx}=xy$.

15. Solve $y''+y=\tan x$ using the method of variation of parameter.

16. Find the Laplace transform of $\frac{1-\cos t}{t^2}$.

17. Find the inverse Laplace transform of $\frac{s^2+2s+5}{s^3}$.

18. Apply convolution theorem to evaluate the inverse Laplace transform of $\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$.

19. Solve $\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}-u=0$ using method of separation of variables.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Solve $x^3y'''-x^2y''+2xy'-2y=\cos(2\log x)$.

21. Expand $f(x)=x\sin x$ as a Fourier series in $0<x<2\pi$.

(1 × 11 = 11 marks)

C 21545–A (Pages : 4) Name.....................................

Reg. No..................................

FOURTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 4B 04—LINEAR ALGEBRA

(Multiple Choice Questions for SDE Candidates)

1. If A and B are square matrices of the same order, then :

   $(AB)^T =$

   (A) $A^T B^T$. (B) $B^T A^T$.

   (C) $A^T + B^T$. (D) $(BA)^T$.

2. A matrix that is both symmetric and upper triangular must be a :

   (A) Diagonal matrix. (B) Non-diagonal but symmetric.

   (C) Both (A) and (B). (D) None of the above.

3. If A and B are invertible matrices with the same size, then AB is invertible and $(AB)^{-1} =$.

   (A) $A^{-1}B^{-1}$. (B) $B^{-1}A^{-1}$.

   (C) Both A and B. (D) None of the above.

4. A matrix E is called ——— if it can be obtained from an identity matrix by performing a single elementary row operation.

   (A) Equivalent matrix. (B) Echelon matrix.

   (C) Elementary matrix. (D) Row reduced matrix.

5. A homogeneous linear system in n unknowns whose corresponding augmented matrix has a reduced row echelon form with r leading 1's has

   (A) n-free variables (B) $n - r$ free variables.

   (C) r-free variables. (D) Cannot be determined.

6. A consistent linear system of two equations in two unknowns has :

   (A) Exactly one solution. (B) Infinitely many solutions.

   (C) Exactly two solutions. (D) Either (A) or (B).

7. If $T_A : R^n \to R^m$ and $T_B : R^n \to R^m$ are matrix transformations, and if $T_A(x) = T_B(x)$ for every vector x in R^n, then :

   (A) (A) and (B) are equivalent but not equal.

   (B) (A) and (B) are equal.

   (C) (A) and (B) cannot be equal.

   (D) Cannot be determined.

8. If W is a subspace of a finite-dimensional vector space V, then :

(A) dim (W) = dim (V) always. (B) dim (W) $\geq$ dim (V).

(C) dim (W) $\leq$ dim (V). (D) None of the above.

9. A $n \times n$ matrix has :

(A) At most n distinct eigenvalues. (B) At least n distinct eigenvalues.

(C) Exactly n distinct eigenvalues. (D) Exactly $n + 1$ distinct eigenvalues.

10. Find the value of m such that the vector $(m, 7, -4)$ is a linear combination of vectors $(-2, 2, 1)$ and $(2, 1, -2)$:

(A) 2. (B) – 2.

(C) 0. (D) – 1.

11. Suppose that $x = (2, 1, 0, 3)$, $y = (3, -1, 5, 2)$. and $z = (-1, 0, 2, 1)$. Which of the following vectors are in *span* $\{x, y, z\}$?

(A) (2, 3, – 7, 3). (B) (1, 1, 1, 1).

(C) Both (A) and (B). (D) Neither (A) nor (B).

12. Which of the following is true ?

(A) A finite set that contains 0 is linearly dependent.

(B) A set with exactly one vector is linearly independent if and only if that vector is not 0.

(C) A set with exactly two vectors is linearly independent if and only if neither vector is a scalar multiple of the other.

(D) All are true.

13. Which of the following sets of vectors in R3 are linearly independent.

(A) $\{(2, 1, 2), (8, 4, 8)\}$.

(B) $\{(1, 1, 0), (1, 1, 1), (0, 1, -1)\}$.

(C) $\{(1, 3, 2), (1, -7, -8), (2, 1, -1)\}$.

(D) $\{(-2, 0, 1), (3, 2, 5), (6, -1, 1), (7, 0, -2)\}$.

14. Transition matrices are :

(A) Not at all invertible. (B) Invertible always.

(C) Invertible sometimes. (D) Data not complete.

15. Let A be any matrix. Then :

(A) $rank\ (A) = rank\ (A^T)$. (B) $rank\ (A) \neq rank\ (A^T)$.

(C) $rank\ (A) < rank\ (A^T)$. (D) $rank\ (A) > rank\ (A^T)$.

16. If A is an $m \times n$ matrix, then :

(A) The null space of A and the row space of A are orthogonal complements in R^n.

(B) The null space of A^T and the column space of A are orthogonal complements in R^m.

(C) Both (A) and (B) are correct.

(D) Neither (A) nor (B) are correct.

17. Let A is an $n \times n$ matrix. The eigenspace of A corresponding to λ is same as :

(A) The null space of the matrix $\lambda I - A$.

(B) The kernel of the matrix operator $T_{\lambda I - A} : R^n \to R^n$

(C) The set of vectors for which $Ax = \lambda x$.

(D) All the above.

18. Let A is an $n \times n$ matrix and suppose A has rank n. Then :

(A) T_A is not one-to-one. (B) $\lambda = 0$ is not an eigenvalue of A.

(C) The range of T_A is $\{0\}$. (D) The kernel of T_A is R^n.

19. Which of the following is true ?

(A) $\langle u, v + w \rangle = \langle u, v \rangle + \langle u, w \rangle$. (B) $\langle u, v + w \rangle = \langle v, u \rangle + \langle w, u \rangle$.

(C) Both (A) and (B) are true. (D) Neither (A) nor (B) is true.

20. Find the correct one from the given statements :

(A) If u is orthogonal to every vector of a subspace W, then $u = 0$.

(B) If u and v are orthogonal, then $|\langle u, v \rangle| \quad \|u\| \, \|v\|$.

(C) If u and v are orthogonal then $\|u + v\| \quad \|u\| \, \|v\|$.

(D) None of these.

C 21545 (Pages : 4) Name..

Reg. No..

FOURTH SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A (Short Answer Type Questions)

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Show that the linear system of equations $\begin{aligned} 4x - 2y &= 1 \\ 16x - 8y &= 4 \end{aligned}$ has infinitely many solutions.

2. Write any two facts about row echelon forms and reduced row echelon forms.

3. Express the linear system
$$\begin{aligned} 4x_1 - 3x_3 + x_4 &= 1 \\ 5x_1 + x_2 - 8x_4 &= 3 \\ 2x_1 - 5x_2 + 9x_3 - x_4 &= 0 \\ 3x_2 - x_3 + 7x_4 &= 2 \end{aligned}$$
in the form AX = B.

4. Let V = R^2 and define addition and scalar multiplication as follows. For $\bar{u} = (u_1, u_2), \bar{v} = (v_1, v_2)$, $\bar{u} + \bar{v} = (u_1 + v_1, u_2 + v_2)$ and for a real number k, $k\bar{u} = (ku_1, 0)$. For $\bar{u} = (1,1)$ and $\bar{v} = (-3,5)$ find $\bar{u} + \bar{v}$ and for $k = 5$, find $k\bar{u}$. Also show that one axiom for vector space is not satisfied.

5. Define basis for a vector space.

6. How will you relate the dimension of a finite dimensional vector space to the dimension of its subspace. Give two facts.

7. Give a solution to the change of basis problem.

8. When you can say that a system of linear equation $Ax = b$ is consistent. What is meant by a particular solution of the consistent system $Ax = b$.

9. Find the rank of a 5 × 7 matrix A for which $Ax = 0$ has a two-dimensional solution space.

10. If $T_A : R^n \to R^m$ is a matrix transformation. Then define its kernel ker (T_A) and Range of (T_A). What is ker (T_A) in terms of null-space of A.

11. Discuss the geometric effect on the unit square of multiplication by a diagonal matrix $A = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$.

12. Confirm by multiplication that x is an eigen vector of A and find the corresponding eigen value, if $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

13. Let R^2 have the weighted Euclidean inner product $< u, v > = 2u_1v_1 + 3u_2v_2$. For $u = (1,1), v = (3,2)$, compute $d(u,v)$.

14. If u and v are orthogonal vectors in a real inner product space, then show that $\| u + v \|^2 = \| u \|^2 + \| v \|^2$.

15. State four properties of orthogonal matrices.

(10 × 3 = 30 marks)

Section B (Paragraph/ Problem Type Questions)

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Suppose that the augmented matrix for a linear system has been reduced to the row echelon form as $\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$ solve the system.

17. If A is an invertible matrix, then show that A^T is also invertible and $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$.

18. Let V be a vector space and $\bar{u}$, a vector in V and k a scalar. Then show that (i) $0\bar{u} = 0$; (ii) $(-1)\bar{u} = -\bar{u}$.

19. If $S = \{v_1, v_2, \ldots, v_n\}$ is a basis for a vector space V, then show that every vector v in V can be expressed in form $v = c_1v_1 + c_2v_2 + \ldots + c_nv_n$ in exactly one way. What are the co-ordinates of V relative to the basis S ?

20. Consider the basis $B = \{u_1, u_2\}$ and $B' = \{u_1', u_2'\}$ for R^2, where $u_1 = (2,2)$ $u_2 = (4,-1)$ $u_1' = (1,3)$ $u_2' = (-1,-1)$.

    (a) Find the transition matrix from B' to B.

    (b) Find the transition matrix from B to B'.

21. If A is a matrix with n columns, then define rank A, nullity of A and establish a relationship between them.

22. Define eigen space corresponding to an eigen value λ of a square matrix A. Also find eigen value and bases for the eigen space of the matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$.

23. Use the Gram–Schmidt process for an orthonormal basis corresponding to the basis vectors $u_1 = (1,1,1)$, $u_2 = (0,1,1)$ and $u_3 = (0,0,1)$.

(5 × 6 = 30 marks)

Section C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. Show that the following statements are equivalent for an $n \times n$ matrix A :

    (a) A is invertible.

    (b) $Ax = 0$ has only the trivial solution.

    (c) The reduced row echelon form of A is I_n.

    (d) A is expressible as a product of elementary matrices.

25. (a) Define Wronskian of the functions $f_1 = f_1(x), f_2 = f_2(x) \ldots f_n = f_n(x)$ which are $n - 1$ times differentiable in $(-\infty, \infty)$. Use this to show that $f_1 = x$ and $f_2 = \sin x$ are linearly independent vectors in $c^{\infty}(-\infty, \infty)$.

    (b) Show that the vectors $v_1 = (1,2,1), v_2 = (2,9,0)$ and $v_3 (3,3,4)$ form a basis for R^3.

26. (a) If A is the matrix $\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$, then find a basis for the row space consisting on entirely row vectors from A.

(b) Find the standard matrix for the operator $T : R^3 \rightarrow R^3$ that first rotates a vector counter clockwise about z-axis through an angle θ, reflects the resulting vector about yz plane and then projects that vector orthogonally onto the xy plane.

27. (a) On P_2, polynomial in $[-1,1]$, define innerproduct as $< p,q > = \int_{-1}^{1} p(x)q(x)dx$. Find $\| p \|, \| q \|$ and $< p,q >$ for $p = x$ and $q = x^2$.

(b) If A is an $n \times n$ matrix with real entries, show that A is orthogonally diagonalizable if and only if A has an orthonormal set of n eigenvectors.

(2 × 10 = 20 marks)

C 21298-A (Pages : 4) Name......................................

Reg. No....................................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2022

Mathematics

ME 4C 04—MATHEMATICAL ECONOMICS

(2014—2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

ME 4C 04—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. r^2 in intercept less model is ———— negative.

   (A) Always. (B) Sometimes.

   (C) Never. (D) Cannot say.

2. In regression through the origin model, ———— is absent.

   (A) The intercept term, β_1. (B) The slope co-efficient, β_2.

   (C) Error term. (D) Explanatory variables.

3. According to Keynes the value of marginal propensity to consume is :

   (A) 0. (B) 1.

   (C) 0 and one. (D) Infinity.

4. In the Keynesian linear consumption function $Y = \beta_1 + \beta_2 X$, Y represents :

   (A) Income. (B) Consumption expenditure.

   (C) Output. (D) Price.

5. In the Keynesian linear consumption function $Y = \beta_1 + \beta_2 X$, the parameters of the model are :

   (A) β_1 and β_2. (B) β_1 and X.

   (C) X and Y. (D) Y and β_2.

6. The term regression was first introduced by :

   (A) Irwing Fisher. (B) Laspayer.

   (C) Francis Galton. (D) Pearson.

7. Regression analysis is concerned with :

   (A) Study of the dependence on one variable on the other.

   (B) Predicting the average value.

   (C) Predicting the population mean.

   (D) All of the above.

8. Statistical relationships assumes that variables are :

(A) Random.

(B) Stochastic.

(C) All of the above.

(D) None of the above.

9. A model in which regressand is linear and regressor is logarithmic is called ————.

(A) Regression through the origin.

(B) Lin log model.

(C) Log lin model.

(D) CLRM.

10. The law of universal regression was first introduced by :

(A) Irwing Fisher.

(B) Laspayer.

(C) Francis Galton.

(D) Pearson.

11. If we are studying the dependence of a variable on more than one explanatory variable, the analysis is called :

(A) Two variable regression analysis.

(B) Multiple regression analysis.

(C) Single regression analysis.

(D) None of the above.

12. The set of all possible outcomes of an experiment or measurement is known as :

(A) Population.

(B) Census.

(C) Sample.

(D) Variable.

13. The locus of points conditional means of the dependent variable for the fixed values of the explanatory variables is :

(A) Venn Diagram.

(B) Lorenz curve.

(C) Probability curve.

(D) Population regression curve.

14. In the regression function $E(Y/X_i) = \beta_1 + \beta_2 X_i$, regression co-efficients are :

(A) Y and X.

(B) Y and β_1.

(C) β_1 and β_2.

(D) β_1 and X_i.

15. Which is the assumption of Gaussian standard classical linear regression model ?

(A) Linear regression model.

(B) X values are fixed.

(C) Zero mean values for disturbances.

(D) All of the above.

16. Homoscedasticity means ———— for disturbances.

(A) Equal mean. (B) Equal variance.

(C) Zero mean. (D) None of the above.

17. In the function, $Q = \alpha + \beta P$, the intercept co-efficient is :

(A) α. (B) β.

(C) P. (D) Q.

18. In the regression context, the OLS estimators are BLUE according to :

(A) Central Limit Theorem. (B) Gauss Markov Theorem.

(C) Young Theorem. (D) Fisher's Theorem.

19. The numerical value of coefficient of correlation lies between :

(A) -1 and 1. (B) 0 and 1.

(C) $-\infty$ to $+\infty$. (D) $-\infty$ to 1.

20. Which of the following is used to measure the degree of association between two variables :

(A) Co-efficient of determination. (B) Co-efficient of correlation.

(C) Standard error. (D) Standard deviation.

C 21298 (Pages : 4) Name.......................................

Reg. No..................................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2022

Mathematics

ME 4C 04—MATHEMATICAL ECONOMCIS

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *marks.*

1. r^2 in intercept less model is ——————— negative.

   A) Always. B) Sometimes.

   C) Never. D) Cannot say.

2. The first step in traditional econometric methodology is :

   A) Statement of theory. B) Forecasting.

   C) Obtaining data. D) Estimation of the model.

3. In the Keynesian linear consumption function $Y = \beta_1 + \beta_2 X$ Y represents :

   A) Income. B) Consumption expenditure.

   C) Output. D) Price.

4. Model in which regressand is logarithmic is called ————————.

   A) Regression through the origin. B) lin log model.

   C) Log lin model. D) CLRM :

5. Stochastic variables are those having :

   A) Probability distribution. B) Indexation.

   C) Correlation. D) Causation.

Turn over

6. The correlation co-efficient between the mathematics and economics was found to be 0.64. What will be the values of correlation co-efficient between economics and mathematics :

A) 0.32.
B) – 0.64.
C) 0.64.
D) 1.28.

7. The locus of points conditional means of the dependent variable for the fixed values of the explanatory variables is :

A) Venn Diagram.
B) Lorenz Curve.
C) Probability curve.
D) Population regression curve.

8. Under the method of OLS, we try to minimise :

A) Residuals.
B) Squared residuals.
C) Co-efficients.
D) Parameters.

9. The sample regression line obtained through the OLS method passes through :

A) Sample means.
B) Sample standard deviation.
C) Origin.
D) Vertical axis.

10. The standard deviation of the sampling distribution of the estimator is called :

A) Standard error.
B) Degree of freedom.
C) Co-efficient level.
D) Test of significance.

11. The summary measure used to measure the goodness of fit of a regression line :

A) Co-efficient of determination.
B) Co-efficient of variation.
C) Standard error.
D) Standard deviation.

12. The accepting of a false hypothesis is called :

A) Type I error.
B) Type II error.
C) Standard error.
D) Point estimation.

(12 × 1 = 12 marks)

Part B

Answer any **six** *questions in two* **or** *three sentences.*
Each question carries 3 *marks.*

13. Write a short note on 'Two Variable Regression Analysis'.

14. What is time series data ?

15. Write three statistical properties of OLS estimators.

16. Show that the sample correlation co-efficient lies between – 1 and 1.

17. If X_1, X_2 and X_3 are uncorrelated variables each having the same standard deviation, show that the co-efficient of correlation between $X_1 + X_2$ and $X_2 + X_3$ is equal to 1/2.

18. Write any *three* properties of OLS estimators under the normality assumption.

19. What do you mean by a confidence interval ?

20. Given the sample regression function $Y_i = \hat{\beta}_2 X_i + \hat{u}_i$. Find $\hat{\beta}_2$.

21. Consider the regression model $y_i = \beta_1 + \beta_2 x_i + u_i$ where $y_i = (Y_i - \bar{Y})$ and $x_i = (X_i - \bar{X})$. In this case, show that the regression line must pass through the origin.

(6 × 3 = 18 marks)

Part C

Answer any **six** *questions from the following.*
Each question carries 5 *marks.*

22. Discuss briefly about the four measurement scales of variables in the regression analysis.

23. Write a note on the Concept of Population Regression Function (PRF).

24. Consider the sample regression $Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$ with the assumptions (i) $\sum \hat{u}_i = 0$; and (ii) $\sum \hat{u}_i X_i = 0$. Obtain the estimators $\hat{\beta}_1$ and $\hat{\beta}_2$.

25. Consider the following formulations of the two-variable PRF : Model I $Y_i = \beta_1 + \beta_2 X_i + u_i$ Model II $Y_i = \alpha_1 + \alpha_2 (X_i - \bar{X}) + u_i$ Find the estimators β_2 and α_2. Are they identical ? Are their variances identical?

26. Find the Confidence Intervals for Regression Co-efficients β_1.

27. Write a note on Testing of Hypothesis.

28. Find $\hat{\beta}_2$ and $\hat{\beta}_1$ from the following data :

X	1	4	5	6
Y	4	5	7	12

29. Consider the loglinear model : $\ln Y_i = \beta_1 + \beta_2 \ln X_i + u_i$ Plot Y on the vertical axis and X on the horizontal axis. Draw the curves showing the relationship between Y and X when $\beta_2 = 1$, and when $\beta_2 > 1$, and when $\beta_2 < 1$.

30. What is Log Linear regression model ? How to measure elasticity using this model.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions from the following.*
Each question carries 10 *marks.*

31. Discuss the significance of the Stochastic Disturbance Term

32. Discuss the assumptions made in the classical linear regression model.

33. Calculate the correlation co-efficient for the following heights (in inches) of fathers(X) and their sons(Y).

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

34. Discuss LogLin and LinLog Models.

(2 × 10 = 20 marks)

C 21297 (Pages : 3) Name.......................................

Reg. No...................................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MAT 4C 04—MATHEMATICS

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Write Laplace transform of $f''(t)$.
2. Write the general form of second order linear ODE.
3. Find L (e^{at}).
4. What is unit step function ? Give an example.
5. Give a formula for an error for Simpson's rule.
6. Find the fundamental period for sin x.
7. Find Wronskian of cos ωx and sin ωx.
8. What is particular solution of an ODE ?
9. What do you mean by an even function give example.
10. Write the 1-dimensional Heat equation.
11. State second shifting theorem for Laplace transfom.
12. Solve $y'' - y = 0$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Find a basis for the solution of the differential equation $y'' - y = 0$.
14. Show that Laplace transform is a linear operator.

Turn over

15. Solve the initial value problem $y'' + 2y' + 2y = 0, y(0) = 1, y'(0) = -1$.

16. Factor $(D^2 + 6D + 13I)y = 0$ and solve it.

17. Find $L^{-1}\left(\dfrac{\sqrt{8}}{\left(s+\sqrt{2}\right)^3}\right)$.

18. If $f(x)$ is a periodic function of x of period p, show that $f(ax), a \neq 0$, is a periodic function of x of period $\dfrac{p}{a}$.

19. Find the Fourier cosine transform of $e^{-ax}, a > 0$.

20. Find an ODE for the basis e^{2x}, e^{x}.

21. Solve $y'' - y = t, y(0) = 1, y'(0) = 1$ by applying Laplace transform.

22. Check whether the functions $5 \sin x \cos x, 3 \sin 2x, x > 0$ are linearly independent.

23. Find solutions u of the PDE $u_{xx} - u = 0$.

24. Find an upper bound for the error incurred in estimating $\int_0^2 5x^4 dx$ using Simpson's rule with $n = 4$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find $L^{-1}\left(\dfrac{1}{\left(s^2+w^2\right)^2}\right)$.

26. Find solution of the initial value problem $y'' + 4y = 16 \cos 2x, y(0) = 0, y'(0) = 0$.

27. Find the Laplace transform of $e^{-at} \cos \beta t$.

28. Find the inverse transform $f(t)$ of $F(s) = \frac{e^{-s}}{s^2 + \pi^2} + \frac{e^{-2s}}{s^2 + \pi^2} + \frac{e^{-3s}}{(s+2)^2}$.

29. Find a general solution of the differential equation :

$y'' + 3y' + 2y = 30e^{2t}$.

30. Find the Fourier series of $f(x) = \begin{cases} -k, \text{ if } -\pi < x < 0 \\ k, \text{ if } 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$.

31. How many subdivisions should be used in the Trapezoidal Rule to approximate $\ln 2 = \int_1^2 \frac{1}{x}\, dx$ with an error whose absolute value is less than 10^{-4}.

32. Given $y' = x(1 - y)$, $y(1) = 0$, $dx = 0.2$. Find the first three approximations by improved Euler method. Compare with exact solution.

33. Evaluate $\int_{-1}^{1} (1 + x^2)\, dx$ with $n = 4$ steps and find an upper bound for $|E_s|$ using Simpson's rule.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve $y'' + 3y' + 2y = 1$ if $0 < t < a$ and 0 if $t > a$; $y(0) = 0$, $y'(0) = 0$.

35. Find the Fourier series of $f(x) = x^2$ in $[-\pi, \pi]$ with $f(x + 2\pi) = f(x)$. Hence deduce that

$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots = \frac{\pi^2}{12}$.

36. Solve the integral equation $y(t) - \int_0^t (1 + \tau) y(t - \tau)\, d\tau = 1 - \sinh t$.

(2 × 10 = 20 marks)

C 21296-A (Pages : 5) Name.......................................

Reg. No....................................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

(2014—2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

(Multiple Choice Questions for SDE Candidates)

1. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$ then $\rho(A)$ is :

   (A) 0.
   (B) 1.
   (C) 2.
   (D) n.

2. The points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are collinear if and only if the rank of the matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ is :

   (A) < 3.
   (B) ≤ 3.
   (C) > 3.
   (D) ≥ 3.

3. If a matrix A can be reduced to the normal form $\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$ by using elementary operations, then $\rho(A)$ is :

   (A) 4.
   (B) 3.
   (C) 2.
   (D) 1.

4. $\text{Rank}(AA') =$ ————————.

   (A) rank A.
   (B) rank A'.
   (C) 1.
   (D) None.

5. For a system of m linear equations in unknowns, Cramer's rule is applicable, when ?

   (A) $m = n$.
   (B) $m \neq n$.
   (C) $m = n$ and the coefficient matrix is non-singular.
   (D) $m = n$ and the coefficient matrix is singular.

6. The system of equations $\begin{matrix} x+2y+z=9 \\ 2x+y+3z=1. \end{matrix}$ can be expressed as :

(A) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$

(B) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 9 \\ 7 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$

(C) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}.$

(D) None.

7. If A is a square matrix of order n and λ is a scalar, then the characteristic polynomial of A is obtained by expanding the determinant.

(A) $|\lambda A|$.

(B) $\lambda|A|$.

(C) $|\lambda_A - I_n|$.

(D) $|A - \lambda I_n|$.

8. The scalar λ is a characteristic root of the matrix A if :

(A) $(A - \lambda I)$ is non singular.

(B) $(A - \lambda I)$ is singular.

(C) A is non singular.

(D) A is singular.

9. If eigen value of matrix A is λ, then eigen value of $p^{-1}AP$ is :

(A) 1.

(B) λ.

(C) $1/\lambda$.

(D) 0.

10. A polynomial equation whose roots are 3 times those of the equation $2x^3 - 5x^2 + 7 = 0$ is :

(A) $3x^3 - 15x^2 + 21 = 0$.

(B) $2x^3 - 15x^2 + 189 = 0$.

(C) $2x^3 + 15x^2 - 189 = 0$.

(D) None.

11. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$ then $\alpha\beta + \beta\gamma + \gamma\alpha$ equals :

(A) $\frac{-p}{q}$. (B) p.

(C) q. (D) $-q$.

12. If I is a unit matrix of order n, then its rank is equal to :

(A) 1. (B) n.

(C) Less than n. (D) Greater than n

13. The rank of the matrix $\begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 2 & 0 \end{bmatrix}$ is :

(A) 3. (B) 4×3.

(C) 2. (D) 1.

14. Rank of the matrix $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}$ is :

(A) 1. (B) 2.

(C) 3. (D) 4.

15. If a matrix A has a nonzero minor of order r, then :

(A) $\rho(A) = r$. (B) $\rho(A) \geq r$.

(C) $\rho(A) < r$. (D) $\rho(A) \leq r$.

16. Which of the following is false ?

(A) $\rho(A+B) \leq \rho(A) + \rho(B)$.

(B) $\rho(A') = \rho(A)$.

(C) $\rho(A+B) = \rho(A) + \rho(B) - 4$, if A and B are matrices of rank z.

(D) $\rho(A-B) \leq \rho(A)\rho(B)$.

17. A system of m non-homogeneous linear equations $Ax = B$ in n unknown is consistant if :

(A) $m = n$.
(B) $m \neq n$.
(C) $\rho(A) \neq \rho([A, B])$
(D) $\rho(A) = \rho([A, B])$

18. A system of m homogeneous linear equations $Ax = 0$ in n unknown has only trivial solution if :

(A) $m = n$.
(B) $m \neq n$.
(C) $\rho(A) = m$.
(D) $\rho(A) = n$.

19. The characteristic roots of skew-Hermition matrix are either :

(A) Real or zero.
(B) Real or non-zero.
(C) Pure imaginary or zero.
(D) Pure imaginary or non-zero.

20. The matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ satisfies the equation :

(A) $A^{\alpha} + 5A + 7I = 0$.
(B) $A^{\alpha} + 5A - 7I = 0$.
(C) $A^{\alpha} - 5A - 7I = 0$.
(D) $A^{\alpha} - 5A + 7I = 0$.

C 21296 (Pages : 4) Name..................................

Reg. No..............................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2022

Mathematics

MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all** *twelve questions.*

Each question carries 1 *mark.*

1. State the fundamental theorem of theory of equations.
2. If α, β, γ are the roots of the equation $ax^3 + bx^2 + cx + d = 0$, write the equation whose roots are $-\alpha, -\beta, -\gamma$.
3. Find the number of real roots of $x^4 - 1 = 0$.
4. Write the standard form of a cubic equation.
5. Find the rank of $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
6. If A and B are non-singular square matrices of order 5, find the rank of AB.
7. Find the number of solutions of the equation $x + 2y = 3$.
8. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 2 \\ a & 6 & b \end{bmatrix}$ and the system of homogeneous linear equations AX = 0 has a non-zero solution, find the value of b.
9. Find the characteristic roots of $\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$.
10. Find the parametric equations of the line through the point (3, – 4, – 1) parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Turn over

11. Find the angle between the planes $x + y = 1$, $2x + y - 2z = 2$.

12. Find the unit tangent vector to the curve $\mathbf{r}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j}$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. If α, β, γ are the roots of the equation $2x^3 + x^2 - 2x - 1 = 0$, find the value of $\alpha + \beta + \gamma$.

14. If α, β, γ are the roots of the equation $2x^3 + 3x^2 - x - 1 = 0$, find the equation whose roots are $\frac{1}{2\alpha}, \frac{1}{2\beta}, \frac{1}{2\gamma}$.

15. Show that the equation $x^4 + 4x^2 + 5x - 6 = 0$ has exactly one positive root.

16. Show that the rank of a matrix, every element of which is unity is 1.

17. Find the normal form of $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \end{bmatrix}$.

18. Find the values of λ so that the system of equations $\lambda x + y = 0$, $x + \lambda y = 0$ has zero solution only.

19. Prove that the characteristic roots of triangular matrix are the same as its diagonal elements.

20. Show that if λ is a characteristic root of a matrix A, then $\lambda + k$ is a characteristic root of the matrix $A + kI$.

21. Find the spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

22. If $\mathbf{r}(t) = (3\cos t)\,\mathbf{i} + (3\sin t)\,\mathbf{j} + t^2\mathbf{k}$ is the position vector of a particle in space at time t, at what times, if any, are the body's velocity and acceleration orthogonal ?

23. If u is a differentiable vector function of t of constant magnitude, prove that $\mathbf{u} \cdot \frac{du}{dt} = 0$.

24. Show that the curvature of a straight line is zero.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Solve $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in A.P.

26. If α, β, γ are the roots of the equation $x^3 + 3x^2 + 6x + 1 = 0$, find the value of $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$.

27. Obtain the real root of the equation $x^3 - 15x = 126$ by Cardan's method.

28. Reducing to the normal form, find the rank of $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

29. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$, find non-singular matrices P and Q such that PAQ is in normal form.

30. Test for consistency and solve the system of equations :

$$5x + 3y + 7z - 4 = 0$$

$$3x + 26y + 2z - 9 = 0$$

$$7x + 2y + 10z - 5 = 0.$$

31. If A is a non-singular matrix, prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A.

32. Find the distance from the point (1, 1, 5) to the line $x = 1 + t, y = 3 - t, z = 2t$.

33. Obtain the curvature of a circle of radius a.

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

35. Find the characteristic roots and the corresponding characteristic vectors for the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

36. Find the binormal vector and torsion for the space curve $\mathbf{r}(t) = (3 \sin t)\,\mathbf{i} + (3 \cos t)\,\mathbf{j} + 4t\mathbf{k}$.

(2 × 10 = 20 marks)