C 20652 (Pages : 4) Name.......................................

Reg. No...................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 14 (E03)—MATHEMATICAL PROGRAMMING WITH PYTHON AND LATEX

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. List any two data types in python. Also define one variable of each type.
2. Write the python code to print the value of the expression $\frac{ab}{\sqrt{a^2 - b^2}}$ by defining the variables $a = 2^{314}$ and $b = \sqrt[3]{10}$.
3. What is mean by slicing of a string ? Give one example.
4. Define a list of elements $3.14, 0, -2, 1+\sqrt{2}, 2^{2^{10}}$ and write the code to print the square of third element.
5. Write python code to input the numbers 100, 2.713 to the variables x, e and print their sum and product.
6. Write python code for plotting the points (0,0), (2,3), (–1, 2) on the plane.
7. Write a function to plot a polar curve.
8. Name any two numerical methods for finding the roots of an equation $f(x) = 0$.
9. Write a python program to print the polynomial $2x + 3$.
10. What is mean by documentclass in $\LaTeX$ Name any two.
11. Write the $\LaTeX$ command to print the following.

    Mathematics, MATHEMATICS

12. Name any two environments in $\LaTeX$ and explain the purpose.

(8 × 3 = 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Write a short note on for and while loops in python. Write a python program to print the square root of natural numbers less than 10,000.

14. Write a short note on the matrix creation and operations on matrices using numpy module. Write a program to print the square of the matrix $A = \begin{pmatrix} 2 & 3 & 4 \\ 0 & 2.5 & 5 \\ 7 & 6 & 3 \end{pmatrix}$.

15. Write python programs to plot the curves :

    (a) $y = x\sin(1/x), 0.001 \le x \le 5$ (use 1000 points).

    (b) $r = \sin 5\theta, 0 \le \theta \le 2\pi$ (use 1000 points).

    (c) $x^2 + y^2 = 1, y \ge 0$.

16. Write a short note on the trapezoidal rule to find $\int_a^b f(x)\,dx$. Write a python program to evaluate $\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx$ using trapezoidal rule with 1000 subintervals of [0,1].

17. Write a short note on Euler's method for solving an initial value problem for first order ordinary differential equations. Write a program to plot the solution curve of the initial value problem $\frac{dy}{dx} = \frac{x\sin x}{1+x^2}, y(0) = 0$. (use step-size equal to 0.001)

18. Write a LaTeX file to produce the following output (without frame).

Derivatives

The derivative of f at c is defined as :

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

if the limit exists.

Find the derivative of the following functions at the given points :

1. $f(x) = \sin x$ at $c = \pi$.
2. $g(x) = x^3 - x^2$ at $c = \frac{1}{\sqrt{2}}$.
3. $h(x) = e^{-\frac{1}{x}}$ at $c = 2^{\pi}$.

19. Write a LaTeX file to produce the following output (without frame).

Hyperbola

The hyperbola with center (h, k) and vertices at $(a, 0)$ and $(b, 0)$ is :

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1.$$

The *foci* are $\left(\sqrt{a^2+b^2}, 0\right)$ and $\left(-\sqrt{a^2+b^2}, 0\right)$. Sketch the following hyperbolas :

- $9y^2 - x^2 = 36$.
- $16(x+1)^2 - 8(y-3)^2 = 16$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Write a short note on conditional statements in python.

(b) Write apython program to input an integer and check whether it is a prime number.

(c) Write a python program to solve the system of equations :

$$2x_1 + 3x_2 - x_3 = 2$$
$$5x_1 - 2.3x_2 + 8x_3 = -7$$
$$10x_1 + 12x_2 - 3x_3 = 1.$$

21. (a) Write a short note on 3D Plots using python.

(b) Write a short note on Runge-Kutta method of fourth order.

(c) Write a python program to solve the initial value problem $\frac{dy}{dx} = \cos x, y(0) = 0$ using Runge-Kutta method of fourth order.

(1 × 11 = 11 marks)

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 14 (E02)—TOPOLOGY OF METRIC SPACES

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Define metric space.
2. Define Isometry between metric spaces.
3. Define Norm on a Linear Space over R or C.
4. Find all limit points of $S = \left\{\frac{1}{n} : n \in N\right\}$ in R with Euclidian metric.
5. Suppose X is a metric space and $A \subseteq B \subseteq X$. Prove that $acc(A) \subseteq acc(B)$.
6. Prove that any function from a Discrete metric space to a metric space is continuous.
7. Suppose X,Y and Z are metric spaces and $f : X \to Y$ and $g : Y \to Z$. If f and $g : Y \to Z$ are continuous, prove that $gof : X \to Z$ is continuous.
8. Suppose X and Y are metric spaces and $f : X \to Y$. Show that f is continuous at every isolated point of X.
9. Define open map. Give one example.
10. Define Connected metric space. Give one example.
11. Find all isolated points of $S = \left\{\frac{1}{n} : n \in N\right\}$ in R with Euclidian metric.
12. Define Cauchy sequence. Give one example.

(8 × 3 = 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Prove that continuous image of a connected metric space is connected.
14. Provethat Every convergent sequence in a metric space is Cauchy. Give one example of a sequence which is not Cauchy
15. Suppose (X, d) be a metric space and $\hat{d}(x,y) = d(x,y)/(1+d(x,y))$ for each $y \in X$. Show that $\hat{d}$ is a metric on X.
16. Suppose X is a metric space and A and B are subsets of X for which $A \subseteq B$. Prove that $diam(A) \leq diam(B)$.
17. Suppose X is a metric space and S is a subset of X. Prove that $\partial S = \partial(S^c)$.
18. Prove that Q, the set of rationals, is dense is R.Give one example of infinite subset of R which is not dense in R.
19. Suppose X is a non-empty metric space, $z \in X$ and $S \subseteq X$. Prove that following statements are equivalent :
    - (i) $diam(S) < \infty$.
    - (ii) There is a ball of X centred at z that includes S.
    - (iii) There is a ball of X that includes S.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Suppose X and Y are metric spaces $f : X \to Y$ and Prove the following statements are equivalent :
    - (i) For each open subset V of Y, the inverse image $f^{-1}(V)$ is open in X.
    - (ii) For each closed subset F of Y, the inverse image $f^{-1}(F)$ is closed .
    - (iii) For each open ball B of Y, the inverse image $f^{-1}(B)$ is open in X.
21. Suppose S is a subset of R. Prove that $diam(S) = \sup S - \inf S$.

(1 × 11 = 11 marks)

C 20650-A

(Pages : 4)

Name.......................................

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 15	Maximum : 15 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 6B 14 (E01)—GRAPH THEORY

(Multiple Choice Questions for SDE Candidates)

1. What is the maximum number of edges in a bipartite graph having 12 vertices ?

   (A) 24.
   (B) 38.
   (C) 36.
   (D) 32.

2. What is the regularity of K_n ?

   (A) n.
   (B) $n - 1$.
   (C) 0.
   (D) 1.

3. A connected planar graph having 6 vertices, 7 edges contains ———— regions.

   (A) 1.
   (B) 3.
   (C) 5.
   (D) 7.

4. For a given graph G having v vertices and e edges which is connected and has no cycles, which of the following statements is true ?

   (A) $v = e$.
   (B) $v = e + 1$.
   (C) $v + 1 = e$.
   (D) $v = e - 1$.

5. A connected undirected graph containing n vertices and $n - 1$ edges ————.

   (A) Cannot have cycles.
   (B) Must contain at least one cycle.
   (C) Can contain atmost two cycles.
   (D) Must contain atleast two cycles.

6. Which of the following statements is/are TRUE for Tree ?

   P : Every two points of G are joined by a unique path.

   Q : G is connected and $p = q - 1$.

   (A) P only.
   (B) Q only.
   (C) Both P and Q.
   (D) Neither P and Q.

7. Let G be a simple graph with every pair of vertices is connected. Then G is :

   (A) Trivial.
   (B) Complete.
   (C) Disconnected.
   (D) Self complementary.

8. Let $G = K_n$ where $n > 5$ .Then number of edges of any induced sub graph of G with 5 vertices :

(A) 10.
(B) 5.
(C) 6.
(D) 8.

9. Number of edges incident with the vertex V is called ?

(A) Degree of a graph.
(B) Handshaking lemma.
(C) Degree of a vertex.
(D) None of the above.

10. A graph with n vertices will definitely have a parallel edge or self loop if the total number of edges are :

(A) Greater than $n - 1$.
(B) Less than $n (n - 1)$.
(C) Greater than $\frac{n(n-1)}{2}$.
(D) Less than $\frac{n^2}{2}$.

11. If G is the forest with 54 vertices and 17 connected components, G has ———— total number of edges.

(A) 37.
(B) 71.
(C) 17.
(D) 54.

12. How many of the following statements are correct ?

1. All cyclic graphs are complete graphs.
2. All complete graphs are cyclic graphs.
3. All paths are bipartite.
4. All cyclic graphs are bipartite.
5. There are cyclic graphs which are complete.

(A) 1.
(B) 2.
(C) 3.
(D) 4.

13. Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to :

(A) 4.
(B) 5.
(C) 6.
(D) 7.

14. Radius of a graph, denoted by rad(G) is defined by ————————.

(A) $\text{Max}\{e(v) : v \in V(G)\}$.

(B) $\text{Min}\{e(v) : v \in V(G)\}$.

(C) $\text{Max}\{d(v,v) : v,u \in V(G)\}$.

(D) $\text{Min}\{d(u,v) : v,u \in V(G)\}$.

15. If a graph is planar, then it is embeddable on a :

(A) Circle.

(B) Square.

(C) Sphere.

(D) Triangle.

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

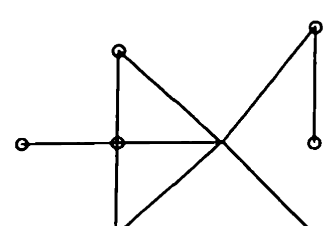
(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A (Short Answer Type Questions)

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Find number of edges of $k_{m,\,n}$.
2. Draw the graph $K_6 - \{v\}$ where v is any vertex in K_6.
3. Draw a 4-regular graph with ten vertices.
4. Define intersection of two graphs.
5. Let G be a simple graph in which there is no pair of adjacent edges. What can you say about the degree of the vertices in G ? Justify.
6. Draw the graph $K_{2,3,3}$.
7. Define eccentricity and radius.
8. Draw Peterson graph and find a trial of length 5.
9. When can you say that the complete graph $kn, n \geq 3$ is Euler ? Justify.
10. Prove that any subgraph of a planar graph is planar.
11. Find K (G) for the graph.



12. How many different Hamiltonian cycles does K_n have ?

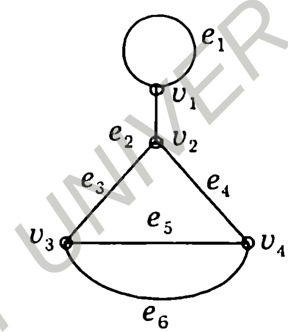
(8 × 3 = 24 marks)

Turn over

Section B (Paragraph/Problem Type Questions)

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. If G is a simple planar graph then prove that G has a vertex v of degree less than 6.
14. Prove that the complete bipartite graph $k_{3,3}$ is non-planar.
15. Let G be a graph with n vertices, where $n \geq 2$. Then prove that G has atleast two vertices which are not cut vertices.
16. Explain the Konigsberg bridge problem.
17. Prove that G is connected if and only if it has a spanning tree.
18. Let G be a graph with n vertices. Then prove that G is a tree if and only if G is a connected graph with $n - 1$ edges.
19. Define (i) adjacency matrix of a graph G ; (ii) incidence matrix of a graph G. Find the adjacency and incidence matrix of the following graph G.



(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. Prove the following :
    (i) A connected graph G has an Euler tail if and only if it is atmost two odd vertices.
    (ii) A simple graph G is Hamiltonian if and only if it closure c (G) is Hamiltonian.
21. Prove that, a non-empty graph with atleast two vertices is bipartite if and only if it has no odd cycle.

(1 × 11 = 11 marks)

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6P 15—RESEARCH METHODOLOGY

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 15**	**Maximum : 15 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 6P 15—RESEARCH METHODOLOGY

(Multiple Choice Questions for SDE Candidates)

1. Choose the false statement.

(A) Inclusion $(\subset)$ is reflexive.
(B) Belonging $(\in)$ is not reflexive.
(C) Inclusion $(\subset)$ is symmetric.
(D) Belonging $(\in)$ is not symmetric.

2. Which of the following is true ?

(A) $\{\phi\} = \phi$.
(B) $\phi \in \{\{\phi\}\}$.
(C) $\{\{\phi\}\} = \{\{\phi\},\{\phi\}\}$.
(D) $\{\phi\} \subset \phi$.

3. "Pure mathematics is, in its way, the poetry of logical ideas." is said by :

(A) Plato.
(B) Mark Twain.
(C) Albert Einstein.
(D) Thomas Storer.

4. PCTeX is related to :

(A) Mathematica.
(B) Maple.
(C) Latex.
(D) Octave.

5. The word research is derived from the French word :

(A) Research.
(B) Recerch.
(C) Resourch.
(D) Riserch.

6. Research is related with :

(A) Discovery of new idea.
(B) Solution of a problem.
(C) Investigation of a problem.
(D) All of the above.

7. Which among the following can be taken as the discrete object ?

(A) People.
(B) Rational numbers.
(C) Integers.
(D) All of the mentioned.

8. Which is correct syntax for LaTeX Table ?

   (A) \begin{tables}.
   (B) \begin{table}.
   (C) \begin{tabular}.
   (D) \begin{tbl}.

9. Syntax for $\supset$.

   (A) \subset.
   (B) \superset.
   (C) \supset.
   (D) None of these.

10. File extension of a LaTeX file :

   (A) .tex
   (B) .doc.
   (C) .latex
   (D) None of these.

11. From a practical perspective, the first thing you need to do before designing an algorithm is ——————.

   (A) Understanding the Problem.
   (B) Analyzing an algorithm.
   (C) Coding an algorithm.
   (D) None of these.

12. Once an algorithm has been specified, you have to prove its ——————.

   (A) Specification.
   (B) Necessity.
   (C) Output.
   (D) Correctness.

13. Name the problem that is to rearrange the items of a given list in nondecreasing order.

   (A) Searching problem.
   (B) Graph problem.
   (C) Sorting problem.
   (D) Numerical problem.

14. How to allign mathematical formula in center ?

   (A) $ .... $.
   (B) $$ ...... $$.
   (C) $ .... $$.
   (D) None of these.

15. What is the output of \Rightarrow ?

   (A) $\Rightarrow$.
   (B) $\rightarrow$.
   (C) $\Leftrightarrow$.
   (D) $\leftrightarrow$.

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6P 15—RESEARCH METHODOLOGY

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. State the axiom of extension. What does the axiom say if A = {1, 2} and B = {1, 2, 3} ?
2. Why is the empty set subset of every set ?
3. State Demorgan's laws.
4. Define a transitive relation. Give an example.
5. For a function $f : X \to Y$, define f^{-1}. If $X = \{-1,1\}, Y = \{-1,1\}, f(x) = x^2$, what is $f^{-1}(-1)$?
6. Give two reasons for learning mathematics.
7. What are two types of mathematical proofs ?
8. What is meant by an algorithm ? Give an example (without details) where an algorithm can be devised.
9. Explain the terms approximation algorithm and exact algorithm.
10. Write the LaTeX code for producing the two sentences :

    Hello world !

    How are you ?
11. Write the LaTeX code for producing the symbols $\in, \subset, \neq, \Rightarrow$.
12. Name two popular Linux distributions.

(8 × 3 = 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Show that (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$; (iii) $A \cap \phi = \phi$.

14. Show that (i) $(A \cup B) \times X = (A \times X) \cup (B \times X)$; (ii) $(A - B) \times X = (A \times X) - (B \times X)$.

15. Give a proof by mathematical induction for the statement: "$5n + 6.7^n + 1$ is divisible by 8 for all. non-negative integers n" .

16. What are the techniques that are a central part of the mathematical thought process ?

17. Explain the consecutive integer checking algorithm for computing gcd (m, n).

18. Give a diagrammatic representation of the design and analysis process of an algorithm.

19. Write a complete LaTeX document to produce the following output given in the box :

1. Let $n = 3$. Then $n^2 + 1 = 10$.
2. The curve $y = \sqrt{x}$, where $x \geq 0$, is concave downward.
3. If $\sin\theta = 0$ and $0 \leq \theta < 2\pi$, then $\theta = 0$ or $\theta = \pi$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Write a note on preparing a mathematical talk giving details on :
    (i) do's and dont's while giving a talk.

    (ii) most important resources available for presenting mathematics.

21. (i) Explain any two problem types in algorithm.
    (ii) Write a short note on two of the differences in storing files in Linux and Windows.

(1 × 11 = 11 marks)

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

11. $L\left[t^{-1/2}\right] =$ ————————.

(A) $-\sqrt{\frac{\pi}{2s}}$.

(B) $\sqrt{\frac{\pi}{2s}}$.

(C) $-\sqrt{\frac{\pi}{s}}$.

(D) $\sqrt{\frac{\pi}{s}}$.

12. The inverse Laplace transform of $\frac{2\pi}{s+\pi}$ is ————————.

(A) $-4\pi e^{-\pi t}$.

(B) $4\pi e^{-\pi t}$.

(C) $-2\pi e^{-\pi t}$.

(D) $2\pi e^{-\pi t}$.

13. $L^{-1}\left[\frac{1}{s^4}\right] =$ ————————.

(A) $\frac{t^3}{6}$.

(B) $\frac{t^3}{4}$.

(C) $-\frac{t^3}{6}$

(D) $\frac{t^3}{10}$.

14. $L^{-1}\left[\frac{s}{s^2+36}\right] =$ ————————.

(A) $-\cos 6t$.

(B) $\cos 6t$.

(C) $\cos 3t$.

(D) $-\cos 3t$.

15. $L^{-1}\left[\frac{5s}{s^2+36}\right] =$ ————————.

(A) $-5\cos 6t$.

(B) $5\cos 6t$.

(C) $-5\sin 6t$.

(D) $5\sin 6t$.

16. The inverse Laplace transform of $\frac{2}{s^2+16}$ is ————————

(A) $\frac{\sin 5t}{4}$.

(B) $\frac{\sin 4t}{5}$

(C) $-\frac{\sin 4t}{4}$.

(D) $\frac{\sin 4t}{4}$.

17. The inverse Laplace transform of $\frac{1}{s(s+1)(s+2)}$ is ——————.

(A) $-\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}$.

(B) $\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}$.

(C) $\frac{1}{2}-e^{-t}-\frac{1}{2}e^{-2t}$.

(D) None of the above options.

18. $L^{-1}\left[\frac{3s+1}{(s-1)(s^2+1)}\right]=$ ——————.

(A) $2e^t-2\cos t+\sin t$.

(B) $-2e^t-2\cos t+\sin t$.

(C) $2e^t-2\cos t-\sin t$.

(D) $2e^t+2\cos t+\sin t$.

19. The inverse transform of $\frac{s+2}{(s+2)^2+1}$ is ——————.

(A) $e^{-2t}\cos t$.

(B) $e^{2t}\cos t$.

(C) $e^{-2t}\sin t$.

(D) $e^{2t}\sin t$.

20. $L[t\sin t]=$ ——————.

(A) $\frac{2s}{(s^2+1)^2}$.

(B) $\frac{2s}{(s^2-1)^2}$.

(C) $-\frac{2s}{(s^2+1)^2}$.

(D) None of the above options.

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find the general solution of the differential equation $\frac{dy}{dt} = -ay + b$ where a,b are positive real numbers.

2. Determine the values of r for which e^{rt} is a solution of the differential equation $y''' - 3y'' + 2y' = 0$.

3. Using method of integrating factors solve the differential equation $\frac{dy}{dt} - 2y = 4 - t$.

4. Find the solution of the differential equation :

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, y(0) = -1.$$

5. Find the Wronskian of the functions $\cos^2\theta, 1 + \cos(2\theta)$.

6. Find the general solution of the differential equation $y'' + 2y' + 2y = 0$.

7. Let $y = \phi(x)$ be a solution of the initial value problem :

$$\left(1 + x^2\right)y'' + 2xy' + 4x^2y = 0, y(0) = 0, y'(0) = 1.$$

Determine $\phi'''(0)$.

8. Determine a lower bound for the radius of convergence of series solutions about each given point $x_0 = 4$ for the given differential equation $y'' + 4y' + 6xy = 0$.

Turn over

9. Find the Laplace transform of the function $\sin(at)$.

10. Find the inverse Laplace transform of $\frac{n!}{(s-a)^{n+1}}$ where $s > a$.

11. Let $u_c(t)$ be unit step function and $L(f(t)) = F(s)$. Show that :

$L(u_c(t)f(t-c)) = e^{cs}F(s)$.

12. Find the inverse Laplace transform of the following function by using the convolution theorem $\frac{1}{s^4(s^2+1)}$.

13. Solve the boundary value problem :

$y'' + y = 0, y(0) = 0, y(\pi) = 0$.

14. Define an even function and show that if $f(x)$ is an even function then :

$\int_{-L}^{L} f(x)dx = 2\int_{0}^{L} f(x)\,dx$.

15. Define the following partial differential equations :

(a) heat conduction equation.

(b) one-dimensional wave equation.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Let $y_1(t)$ be a solution of $y' + p(t)y = 0$ and let $y_2(t)$ be a solution of $y' + p(t)y = g(t)$.

Show that $y(t) = y_1(t) + y_2(t)$ is also a solution of equation $y' + p(t)y = g(t)$.

17. Find the value of b for which the following equation is exact, and then solve it using that value of b.

$(xy^2 + bx^2y) + (x+y)x^2y' = 0$.

18. Solve the initial value problem

$y'' + 4y = t^2 + 3e^t, y(0) = 0, y'(0) = 2$.

19. Use method of variation of parameters find the general solution of :

$$y'' + 4y = 8\tan t, -\pi/2 < t < \pi/2.$$

20. Find the solution of the initial value problem :

$$2y'' + y' + 2y = \delta(t-5), y(0) = 0, y'(0) = 0.$$

here $\delta(t)$ denote the unit impulse function.

21. Using Laplace transform solve the initial value problem :

$$y'' + 4y = 0,\ y(0) = 3,\ y'(0) = -1.$$

22. Find the co-efficients in the Fourier series for f:

$$f(x) = \begin{cases} 0, & -3 < x < -1 \\ 1, & -1 < x < 1 \\ 0, & 1 < x < 3 \end{cases}$$

Also suppose that $f(x + 6) = f(x)$.

23. Find the solution of the following heat conduction problem :

$$100u_{xx} = u_t, 0 < x < 1, t > 0$$
$$u(0,t) = 0, u(1,t) = 0, t > 0$$
$$u(x,0) = \sin(2\pi x) - \sin(5\pi x), 0 \le x \le 1.$$

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. Find the general solution of the following differential equation using the method of integrating factors :

$$\frac{dy}{dt} + \frac{1}{2}y = \frac{1}{2}e^{t/3}.$$

Draw some representative integral curves of the differential equation and also find the particular solution whose graph contains the point (0,1).

25. Find a series solution of the differential equation :

$$y'' + y = 0, -\infty < x < \infty.$$

26. Find the Laplace transform of $\int_0^t \sin(t-\tau)\cos\tau \; d\tau$

27. Find the temperature $u(x, t)$ at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20°C throughout and whose ends are maintained at 0°C for all $t > 0$.

(2 × 10 = 20 marks)

C 20647-A

(Pages : 4)

Name.......................................

Reg. No...................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS-UG)

Mathematics

MTS 6B 12—CALCULUS OF MULTIVARIABLE

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 6B 12—CALCULUS OF MULTIVARIABLE

(Multiple Choice Questions for SDE Candidates)

1. If $f(x,y) \geq g(x,y)$ on D, then $\iint_D f(x,y)\,dA \ldots$

   (A) $\leq \iint_D g(x,y)\,dA$. (B) $\geq \iint_D g(x,y)\,dA$.

   (C) 0. (D) None of these.

2. Evaluate $\iint_R \left(1-2xy^2\right) dA$, where $R = \{(x,y) \mid 0 \leq x \leq 2, -1 \leq y \leq 1\}$:

   (A) $\frac{4}{3}$. (B) $\frac{1}{3}$

   (C) 1. (D) 0.

3. Evaluate $\iiint_T x\,dV$, where T is the part of the region in the first octant lying inside the sphere $x^2+y^2+z^2=1$.

   (A) $\frac{\pi}{6}$. (B) $\frac{\pi}{16}$.

   (C) $\frac{\pi}{4}$. (D) π.

4. Evaluate $\lim_{(x,y)\to(1,2)} \left(x^3y^2 - x^2y + x^2 - 2x + 3y\right)$:

   (A) 0. (B) 1.

   (C) 7. (D) -7.

5. Let f be a function that is defined for all points (x, y) close to the point (a, b). Then f is continuous at the point (a, b) if :

   (A) $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$. (B) $\lim_{(x,y)\to(a,b)} f(x,y)$ exists.

   (C) $\lim_{(x,y)\to(a,b)} f(x,y)$ dos not exists. (D) $\lim_{(x,y)\to(a,b)} f(x,y) = f(0,0)$.

6. A region R is said to be an open region if every point of R is a/an ————.

   (A) Cluster point. (B) Interior point of R.

   (C) Closure of R. (D) None of these.

7. Determine where the function is continuous : $f(x,y) = \frac{xy\left(x^2-y^2\right)}{x^2+y^2}$:

   (A) Everywhere except at (0,0). (B) Everywhere.

   (C) Only at the center. (D) At (1,1).

8. Evaluate $\lim\limits_{(x,y,z)\to\left(\frac{\pi}{2},0,1\right)} \dfrac{e^{2}y(\sin x + \cos y)}{1+y^{2}+z^{2}}$

(A) 0.

(B) 1.

(C) 2.

(D) −2.

9. Find $\dfrac{\partial f}{\partial x}$ if $f(x,y) = 2x^2y^3 - 3xy^2 + 2x^2 + 3y^2 + 1$:

(A) $6x^2y^3 - 3xy^2 + 4x^2 + 3y^2$.

(B) $4xy^3 - 3y^2 + 4x + 3y^2$.

(C) $6x^2y^2 - 6xy + 6y$.

(D) 0.

10. Suppose z is a differentiable function of x and y that is defined implicitly by $x^2 + y^3 - z + 2yz^2 = 5$. Find $\partial z/\partial x$.

(A) $\dfrac{2}{1-4yz}$.

(B) $\dfrac{2x}{1-4yz}$.

(C) $\dfrac{2x}{4yz}$.

(D) None of these.

11. The function $(x,y) = e^x \cos y$ is ———— in the xy-plane.

(A) Real.

(B) Harmonic.

(C) Conjugate.

(D) Complex.

12. Let $w = 2x^2y$, where $x = u^2 + v^2$ and $y = u^2 - v^2$. Find $\partial w/\partial v$.

(A) $4xu\ (2y + x)$.

(B) $2xu\ (2y + x)$.

(C) $4xu\ (2y - x)$.

(D) $2xv\ (2y + x)$.

13. Find $\dfrac{\partial z}{\partial y}$ if $2x^2z - 3xy^2 + yz - 8 = 0$.

(A) $\dfrac{6xy - z}{2x^2 + y}$.

(B) $\dfrac{3y^2 - 4xz}{2x^2 + y}$.

(C) $-\dfrac{3x^2 + y}{x + 2y}$.

(D) $-\dfrac{3x^2 + z}{x + 2y}$.

14. Compute $\iint_S F \cdot n dS$ given that $F(x,y,z) = (x + \sin z)i + (2y + \cos x)j + (3z + \tan y)k$ and S is the unit sphere $x^2 + y^2 + z^2 = 1$.

(A) 8π. (B) 4π.

(C) 2π. (D) π.

15. If $\nabla f(x,y) = 0$, then $D_u f(x,y)$ ———— for every u.

(A) $= 0$. (B) > 0.

(C) < 0. (D) $= 1$.

16. Let f be continuous on a polar rectangle $R = \{(r,\theta) \mid 0 \le a \le r, \alpha \le \theta \le \beta\}$, where $0 \le \beta - \alpha \le 2\pi$. Then $\iint_R f(x,y)\, dA =$ ————.

(A) $\int_\alpha^\beta \int_a^b f(\cos\theta, \sin\theta)\, r\, dr\, d\theta$. (B) $\int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\, dr\, d\theta$.

(C) $\int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$. (D) $-\int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$.

17. The Jacobian of the transformation T defined by $x = g(u,v)$ and $y = h(u,v)$ is :

(A) $\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$. (B) $\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$.

(C) $-\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$. (D) $-\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$.

18. If $w = f(x^2 - y^2, y^2 - x^2)$ and f is differentiable, then $y\frac{\partial w}{\partial x} + x\frac{\partial w}{\partial y} =$ ————.

(A) 2. (B) 0.

(C) –1. (D) 1.

19. Let f be a function of two variables x and y. The gradient of f is the vector function $\nabla f(x,y) =$ ————

(A) $f_y(x,y)i + f_x(x,y)j$. (B) $f_x(x,y)j + f_y(x,y)i$.

(C) $f_x(x,y)i + f_y(x,y)j$. (D) $f_x(x,y)i + f_y(x,y)k$.

20. Suppose that f has continuous second-order partial derivatives on an open region containing a critical point (a, b) of f. Let $D(x,y) = f_{xx}(x,y) f_{yy}(x,y) - f_{xy}^2(x,y)$. If $D(a, b) > 0$ and $f_{w}(a, b) < 0$, then $f(a, b)$ is a ————.

(A) Relative minimum value. (B) Saddle point.

(C) Relative maximum value. (D) None of these.

C 20647 (Pages : 3) Name.......................................

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 12—CALCULUS OF MULTIVARIABLE

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A (Short Answer Questions)

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find the domain and rang of the function $f(x,y) = x + 3y - 1$.

2. Evaluate $\lim_{(x,y,z)\to\left(\frac{\pi}{2},0,1\right)} \frac{e^{2y}(\sin x + \cos y)}{1 + y^2 + z^2}$.

3. Find f_x and f_y if $f(x,y) = x\cos xy^2$.

4. Find the directional derivative of $f(x,y) = x^2 \sin 2y$ at $\left(1, \frac{\pi}{2}\right)$ in the direction of $\bar{v} = 3\hat{i} - 4\hat{j}$.

5. Find $\nabla f(x,y,z)$ if $f(x,y,z) = x^2 + y^2 - 4z$ and find the direction of maximum increase of f at the point (2, –1, 1).

6. Find the relative extrema of the function $f(x,y) = x^2 + y^2 - 2x + 4y$.

7. Find the volume of the solid lying under the elliptic paraboloid $z = 8 - 2x^2 - y^2$ above the rectangular region given by $0 \le x \le 1, 0 \le y \le 2$.

8. Find the mass of the triangular lamina with vertices $(0,0),(0,3)$ and $(2,3)$ given that the density at (x, y) is $\rho(x,y) = 2x + y$.

Turn over

9. Find the Jacobian for the change of variables defined by $x = r\cos\theta, y = r\sin\theta$.

10. Evaluate $\iint_R 2x - y \, dA$ where R is the region bounded by the parabola $x = y^2$ and the line $x - y = 2$.

11. Find whether the vector field $\bar{F} = x^2 y \, \hat{i} + xy \, \hat{j}$ is conservative.

12. State Green's theorem.

13. Find a parametric representation for the cone $x = \sqrt{y^2 + z^2}$.

14. Find the surface area of the torus given by $\bar{r}(u,v) = (2 + \cos u)\cos v \, \hat{i} + (2 + \cos u)\sin v \, \hat{j} + \sin u \, \hat{k}$ where the Domain D is given by $0 \le u \le 2\pi$ and $0 \le v \le 2\pi$.

15. Compute $\iint_S \bar{F} \cdot \hat{n} \, dS$ given $\bar{F}(x,y,z) = (x + \sin z)\,\hat{i} + (2y + \cos x)\,\hat{j} + (3z + \tan y)\hat{k}$ and S is the unit sphere $x^2 + y^2 + z^2 = 1$.

(10 × 3 = 30 marks)

Section B (Paragraph Questions)

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Find f_{xyx} and f_{yxy} if $f(x,y) = x\cos y + y\sin x$.

17. Find the differential of $w = x^2 + xy + z^2$. Compute the value of dw if (x, y, z) changes from (1, 2, 1) to $(0.98, 2.03, 1.01)$ and compare the value with that of Δw.

18. Find the equation of the tangent plane and normal line to the surface $x^2 - 2y^2 - 4z^2 = 4$ at (4, –2, –1).

19. Find the relative extrema of $f(x,y) = -x^3 + 4xy - 2y^2 + 1$.

20. Find the volume of the solid region bounded by the paraboloid $z = 4 - x^2 - 2y^2$ and the xy-plane.

21. Evaluate $\int_0^2 \int_0^x \int_0^{x+y} e^x (y+2z)\, dz\, dy\, dx$.

22. Determine whether the vector field $\vec{F} = e^x (\cos y\, \hat{i} - \sin y\, \hat{j})$ is conservative. If so, find a potential function for the vector field.

23. Evaluate $\oint_C (e^x + y^2)\, dx + (x^2 + 3xy)\, dy$, where C is the positively oriented closed curve lying on the boundary of the semi annular region R bounded by the upper semicircles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ and the x-axis.

(5 × 6 = 30 marks)

Section C (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Sketch the graph of $f(x,y) = 9 - x^2 - y^2$.

(b) Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$ does not exist.

25. (a) Find the relative extrema of $f(x,y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.

(b) Find the minimum value of $f(x,y,z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y - 4z = 49$.

26. Let R be the region bounded by the square with vertices $(0,1), (1,2), (2,1)$ and $(1,0)$. Evaluate $\iint_R (x+y)^2 \sin^2(x-y)\, dA$.

27. Let $\vec{F} = (x,y,z) = 2xyz^2\, \hat{i} + x^2z^2\, \hat{j} + 2x^2yz\, \hat{k}$.

(a) Show that $\vec{F}$ is conservative and find a scalar function f such that $\vec{F} = \nabla f$.

(b) If $\vec{F}$ is a force field, find the work done by $\vec{F}$ in moving a particle along any path from (0, 1, 0) to (1, 2, −1).

(2 × 10 = 20 marks)

C 20646-A (Pages : 4) Name..................................

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 6B 11—COMPLEX ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. Which of the following function $f(z)$, of the complex variable z, is not analytic at all the points of the complex plane ?

   (A) $f(z) = z^2$. (B) $f(z) = e^z$.

   (C) $f(z) = \sin z$. (D) $f(z) = \log z$.

2. If z is a non-zero complex number, then for $n = 1, 2, 3, \ldots$, $z^{1/n}$ is :

   (A) $\exp(n \log z)$. (B) $\exp\left(\frac{1}{n}\log z\right)$.

   (C) $\exp\left(\frac{1}{n}\log\frac{1}{z}\right)$. (D) $\exp\left(n\log\frac{1}{z}\right)$.

3. Real part of the function $|z|^2$ is :

   (A) $x^2 - y^2$. (B) $2xy$.

   (C) $x^2 + y^2$. (D) $y^2 - x^2$.

4. If a function f is analytic throughout a simple connected domain D, then $\int_C f(z)\, dz =$ ————.

   (A) 0. (B) $2\pi i$.

   (C) $2\pi i f(z)$. (D) 1.

5. The integral of $\int_C \frac{dz}{z-i}$ where C is the circle $|z| = 2$ is :

   (A) $2\pi i$. (B) $-\pi i$.

   (C) πi. (D) $-2\pi i$.

6. The integral of the function $\int_C \frac{e^z}{z} dz$, where C is the unit circle is :

   (A) $2\pi i$. (B) $-\pi i$.

   (C) πi. (D) $-2\pi i C$.

7. The integral of the function $\int_C \frac{z+2}{z-2} dx$ where C is the unit circle is :

   (A) 0. (B) 1.

   (C) -1. (D) π.

8. If f is continuous in a domain D and if $\int_C f(z)dz = 0$ for every simple closed positively oriented contour C in D, then :

(A) f is analytic in D.
(B) f is real valued in D.
(C) f is constant in D.
(D) f is imaginary in D.

9. Piecewise smooth curve is also known as :

(A) Contour.
(B) Smooth curve.
(C) Circle.
(D) Regular curve.

10. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ is :

(A) 1.
(B) 0.
(C) –1.
(D) ∞.

11. The center of the power series $\sum_{n=0}^{\infty} (z+4i)^n$ is :

(A) $4i$.
(B) $2i$.
(C) 4.
(D) $-4i$.

12. The zero of the function $\frac{z}{\cos z}$ is :

(A) 1.
(B) 0.
(C) –1.
(D) π.

13. The principal part of $f(z)$ at z_0 consists of infinite number of terms, then z_0 is known as :

(A) Pole.
(B) Essential singular point.
(C) Removable singular point.
(D) Simple pole.

14. The power series $b_0 + b_1^{z-1} + b_2^{z-2} + \ldots$ converges.

(A) Inside of some circle $|z| = R$.
(B) On the circle $|z| = 1$.
(C) On some circle $|z| = R$.
(D) Outside of some circle $|z| = R$.

15. The residue of the function $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z = 3$ is :

(A) -8.

(B) $\frac{101}{16}$.

(C) 0.

(D) $\frac{27}{16}$.

16. Residue of the function $\frac{4}{1-z}$ at the singular points is :

(A) 4.

(B) -4.

(C) 2.

(D) -2.

17. Given $f(z) = \frac{z^2}{z^2 + a^2}$. Then :

(A) $z = ia$ is a pole and $\frac{ia}{2}$ is a residueat $z = ia$ of $f(z)$.

(B) $z = ia$ is a simple pole and ia is a residue at $z = ia$ of $f(z)$.

(C) $z = ia$ is a simple pole and $-\frac{ia}{2}$ is a residue at $z = ia$ of $f(z)$.

(D) None of the above.

18. The value of $\oint \frac{1}{z^2} dz$, where the contour is the unit circle traversed clockwise is :

(A) $-2\pi i$.

(B) 0.

(C) $2\pi i$.

(D) $4\pi i$.

19. The integral of $\int_C \frac{1}{z \sin z} dz$ where C is the unit circle oriented in the positive direction is :

(A) πi.

(B) $2\pi i$.

(C) $-\pi i$.

(D) 0.

20. The zero of order is known as :

(A) Complex zero.

(B) Simple zero.

(C) Singularity.

(D) None of these.

C 20646 (Pages : 2) Name.....................................

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS-UG)

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Define holomorphic function in a domain D. And give an example for an entire function.

2. Prove or disprove : if f is differentiable a point z_0, then f is continuous at that point.

3. Define harmonic function with example.

4. Prove that $\sin^2 z + \cos^2 z = 1$.

5. State ML inequality.

6. Define the path independence for a contour integral.

7. State maximum modulus theorem.

8. Prove that $\int_a^b f(t)\,dt = -\int_b^a f(t)\,dt$.

9. Prove or disprove if $\lim_{n\to\infty} z_n = 0$, then $\sum_{k=1}^{\infty} z_k$ converges.

10. Find the radius of convergence of $\sum_{k=1}^{\infty} \frac{z^k}{k}$.

11. Define pole of order n. Give an example of a function with simple pole at $z = 1$.

12. Find the principal part in the Laurent series expansion about the origin of the function $f(z) = \frac{\sin z}{z^4}$.

Turn over

13. State Rouche's theorem.

14. Find the residue of $\frac{\sin z}{z}$ at $z = 0$.

15. How many zeroes of are in the disc $|z| = 1$ for the function $f(z) = z^9 - 8z^2 + 5$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Check whether the function U is harmonic or not if so find its harmonic conjugate $U(x, y) = x^3 - 3xy^2 - 5y$.

17. Find all the solutions of the equation $\sin z = 5$.

18. State and prove Fundamental theorem of algebra.

19. State and prove Morera's theorem.

20. Find the Taylor's series expansion with centre $z_0 = 2i$ of $f(z) = \frac{1}{1-z}$.

21. Identify the singular points and classify them $f(z) = \frac{\sin z e^{\left(\frac{1}{z-1}\right)}}{z(1+z)}$.

22. Find residue of $e^{e^{\left(\frac{1}{z}\right)}}$ at $z = 0$.

23. Find $\int_{-\infty}^{\infty} \frac{1}{x^4 + 1} dx$.

(5 × 6 = 30 marks)

Section C (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. State and prove Cauchy Riemann Equation. Also state the sufficient condition for differentiability.

25. State and prove Cauchy's integral formula for derivatives.

26. Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for $1 < |z - 2| < 2$.

27. State and prove Cauchy's residue theorem.

(2 × 10 = 20 marks)

C 20645-A

(Pages : 6)

Name.......................................

Reg. No....................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS-UG)

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 6B 10—REAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. Which of the following are true about Thomae's function ?

   (A) Continuous only at irrationals.
   (B) Continuous every where on $\mathbb{R}$.
   (C) Continuous only at rationals.
   (D) Discontinuous only at irrationals.

2. $g : A \to \mathbb{R}$, where $A \subseteq \mathbb{R}$, is uniformly continuous function when, :

   (A) $A = (0, 1)$.
   (B) $A = (-1, 0)$.
   (C) $A = [a, 1)$; $a > 0$.
   (D) $A = (0, \infty)$.

3. Let f and g be *two* Lipschitz functions on $A \subset \mathbb{R}$. Choose the false statement from below :

   (A) cf is a Lipschitz function on A where c is a constant.
   (B) $f + g$ is Lipschitz function on A.
   (C) f^2 is a Lipschitz function on A.
   (D) $f - g$ is a Lipschitz function on A.

4. Let $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \dfrac{1}{1 + x^2}$ then,

   (A) f is uniformly continuous on $\mathbb{R}$.
   (B) f is not uniformly continuous on $\mathbb{R}$.
   (C) f is uniformly continuous only on bounded subset of $\mathbb{R}$.
   (D) f is uniformly continuous only on a closed bounded subset of $\mathbb{R}$.

5. If $0 < x < 1$, which of the following terms increases as x increases :

(i) $1 - x^3$.

(ii) $x \quad 1$.

(iii) $\frac{1}{x^2}$.

(A) (i) only. (B) (ii) only.

(C) (iii) only. (D) (i) and (iii).

6. Evaluate $\lim_{x \to 0} \frac{\log x}{\cot x}$.

(A) 0. (B) 1.

(C) 2. (D) 0.5.

7. Let $f(x) = g(x)$ for $[0,1] \setminus \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$. Choose the false statement from below :

(A) If f is Riemann integrable on [0, 1] then g also Riemann integrable on [0, 1].

(B) If g is Riemann integrable on [0, 1] the f also Riemann integrable on [0, 1].

(C) $\int f dx = \int g dx$.

(D) If f is Riemann integrable on [0, 1] then $\int f dx = \int g dx$.

8. Which of the following is true ?

(A) There exists $f \in \mathcal{R}[a, b]$, such that f is not bounded on $[a, b]$.

(B) There exists $f \in \mathcal{R}[a, b]$, such that f is unbounded on $[a, b]$.

(C) Unbounded functions on $[a, b]$ is integrable when functions takes only positive values.

(D) If f is not bounded on $[a, b]$ then $f \notin \mathcal{R}[a, b]$.

9. Let H $(x) := k$ for $x = 1/k \left(k \in \mathbb{N}\right)$ H $(x) := 0$ elsewhere on [0, 1]. H is not Riemann integrable.

(A) $H \notin \mathcal{R}[0, 1]$.

(B) $\int_0^1 H = 0$.

(C) $\int_0^1 H = 1$.

(D) $\int_0^1 H = \frac{1}{2}$.

10. Choose the false statement from below :

(A) Any subset of a null set is also a null set.

(B) Union of two null set is a null set.

(C) Countable set is a null set.

(D) Every null set is a countable set.

11. Let $g_n(x) := x^n$ for $x \in \mathbb{R}, n \in \mathbb{N}$ then :

(A) (g_n) converges to a continuous function on (–1, 1].

(B) (g_n) converges to a function on (–2, 1].

(C) (g_n) converges to a continuous function on (0, 1].

(D) (g_n) converges to a continuous function on (– 1, 1).

12. For $x \in \mathbb{R}$, $\lim \left(\left(1/n\right) \sin\left(nx + n\right)\right) = ?$

(A) – 1.

(B) 0.

(C) 1.

(D) 2.

13. Let (f_n) converges uniformly on $\mathbb{R}$ to f, then :

(A) $(2f_n)$ does not converge uniformly on $\mathbb{R}$.

(B) (f_n^2) converge uniformly on $\mathbb{R}$.

(C) (f_n^3) converge uniformly on $\mathbb{R}$.

(D) None of these.

14. Find $\sum_{n=0}^{\infty} \frac{1}{(\alpha+n)(\alpha+n+1)}$ if $\alpha > 0$.

(A) $\frac{1}{\alpha}$.

(B) 0.

(C) $-\frac{1}{\alpha}$.

(D) 2α.

15. Choose the correct statement :

(A) $\sum_{n=1}^{\infty} \cos n$ is divergent and the series $\sum_{n=1}^{\infty} (\cos n)/n^2$ is convergent.

(B) $\sum_{n=1}^{\infty} \cos n$ is divergent and the series $\sum_{n=1}^{\infty} (\cos n)/n^2$ is divergent.

(C) $\sum_{n=1}^{\infty} \cos n$ is convergent and the series $\sum_{n=1}^{\infty} (\cos n)/n^2$ is convergent.

(D) $\sum_{n=1}^{\infty} \cos n$ is convergent and the series $\sum_{n=1}^{\infty} (\cos n)/n^2$ is divergent.

16. Find $\int_{-\infty}^{0} e^x \, dx$:

(A) 1.

(B) 0.

(C) – 1.

(D) – 2.

17. Find $\int_{-\infty}^{0} x^2 \, dx$.

(A) 1.

(B) ∞.

(C) 0.

(D) None of these.

18. Find P.V. $\int_{-\infty}^{\infty} x dx$:

(A) 1.

(B) ∞.

(C) 0.

(D) $-\infty$.

19. Find P.V. $\int_{-\infty}^{\infty} x^2 dx$:

(A) 1.

(B) ∞.

(C) 0.

(D) $-\infty$.

20. Find P.V. $\int_{-\infty}^{\infty} \frac{dx}{x^2+1}$:

(A) π.

(B) $\frac{\pi}{2}$.

(C) 0.

(D) $-\pi$.

C 20645 (Pages : 3) Name..

Reg. No.....................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Define continuity of a function. Show that the constant function $f(x)=b$ is continuous on $\mathbb{R}$.

2. State Boundedness theorem. Is boundedness of the interval, a necessary condition in the theorem ? Justify with an example.

3. If $f : A \to IR$ is uniformly continuous on $A \subseteq \mathbb{R}$ and (x_n) is a Cauchy sequence in A. Then show that $f(x_n)$ is a Caychy sequence in $\mathbb{R}$.

4. Define Riemann sum of a function $f:[a,b]\to\mathbb{R}$.

5. Suppose f and g are in $\mathbb{R}[a,b]$ then show that $f+g$ is in $\mathbb{R}[a,b]$.

6. State squeeze theorem for Riemann integrable functions.

7. If f belong to $\mathbb{R}[a,b]$, then show that its absolute value $|f|$ is in $\mathbb{R}[a,b]$.

8. Define pointwise convergence of a sequence (f_n) of functions.

9. Discuss the uniform convergence of $f_n(x)=x^n$ on $(-1,1]$.

10. If $h_n(x)=2nxe^{-nx^2}$ for $x\in[0,1], n\in\mathbb{N}$ and $h(x)=0$ for all $x\in[0,1]$, then show that :

$$\lim\int_0^1 h_n(x)\,dx \neq \int_0^1 h(x)\,dx.$$

11. State Cauchy criteria for uniform convergence series of functions.

12. Evaluate $\int_{-1}^{0} \frac{dx}{\sqrt[3]{x}}$.

13. What is Cauchy principle value. Find the principal value of $\int_{1}^{1} \frac{dx}{x}$.

14. State Leibniz rule for differentiation of Ramann integrals.

15. State that $\Gamma(p+1) = p\Gamma p$ for $p > 0$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Show that the Dirichlet's function :

$$f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}$$ is not continuous at any point of $\mathbb{R}$.

17. State and prove Bolzano intermediate value theorem.

18. Show that the following functions are not uniformly continuous on the given sets :

(a) $f(x) = x^2$ on $A = [0, \infty]$.

(b) $g(x) = \sin \frac{1}{x}$ on $B = (0, \infty)$.

19. If $f : [a,b] \to \mathbb{R}$ is continuous on $[a,b]$, then show that $f \in \mathbb{R}[a,b]$.

20. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$. Then show that f is continuous on A.

21. Let $f_n : [0,1] \to \mathbb{R}$ be defined for $n \geq 2$ by :

$$f_n(x) = \begin{cases} n^2 x & , 0 \leq x \leq \frac{1}{n} \\ -n^2(x - 2/n), & \frac{1}{n} \leq x \leq \frac{2}{n} \\ 0 & , \frac{2}{n} \leq x \leq 1. \end{cases}$$

Show that the limit function is Riemann integrable. Verify whether $\lim \int_{0}^{1} f_n(x) = \int_{0}^{1} f(x)\,dx$.

22. Given $\iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\,dx\,dy = \pi$, find the value of $\int_0^{\infty} e^{-x^2}dx = \sqrt{\frac{\pi}{2}}$.

23. Show that $\forall p > 0, q > 0$ $B(p,q) = \frac{\Gamma p \Gamma q}{\Gamma(p+q)}$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. State and prove Location of roots theorem.

25. State and prove Additivity theorem.

26. Evaluate (a) $\lim \frac{x^n}{1+x^n}$ for $x \in \mathbb{R}, x \geq 0$. (b) $\lim \frac{\sin nx}{1+nx}$ for $x \in \mathbb{R}, x \geq 0$.

Discuss about their uniform convergence.

27. (a) Show that $\forall q > -1, \int_0^1 x^q e^{-x} dx$ converges.

(b) Show that $\forall q \leq -1, \int_0^1 x^q e^{-x} dx$ diverges.

(2 × 10 = 20 marks)

C 20213 (Pages : 3) Name.................................

Reg. No.................................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 13 [E03]—C PROGRAMMING FOR MATHEMATICAL COMPUTING

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. C programming language is developed by ————.
2. What is the meaning of the constant \v in C ?
3. ———— is an example of a logical operator in C.
4. What is the output of the program ?

```
#include<stdio.h>
#define sqr(i) i*i
main()
{printf("% %d", sqr(3), sqr(3+1));}
```

5. What is the difference between a declaration and a definition of a variable in C ?
6. ———— command is used to skip a part of a loop.
7. Write the syntax of *do* statement.
8. Find errors, if any of the statement :

```
if(x + y=z && y > 0)
printf("  ");
```

9. What is the output of the program :

```
#include ¡stdio.h¿
main()
{int a = 3, b = 2; int c = a + b;
printf("%d", &c);}
```

10. What is the general form of declaration of a string variable ?
11. Which of the following is not a storage class specifier in C ?

(a) auto ; (b) register ; (c) static ; (d) extern ; (e) volatile.

12. Which one of the following variable names is NOT valid ?

    (a) go_cart ; (b) go4it ; (c) 4sea ; (d) run6 ; (e) All the above.

(12 × 1 = 12 marks)

Section B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. What is the significance of program algorithms ?
14. What are the different steps involved in the execution of C programs ?
15. What are the basic data types associated with C ?
16. Specify the relational operators and their meaning in C.
17. Explain the working of conditional operator in C.
18. What is header file in C ?
19. What do you mean by a garbage value in a variable ?
20. Write a simple C program to find the maximum of three numbers.
21. What is the difference between scanf and getchar ?
22. What is a null statement ? Explain its usefulness.
23. What do you mean by chaining of functions and recursion of functions ?
24. Distinguish between actual and formal arguments.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. What is a flow chart ? What are the advantages and drawbacks of flow charts ?
26. What are the rules for identifiers in C ?
27. Explain switch statement in C with syntax and suitable example.
28. Explain the working of *for* loop in C.
29. Write a C program to reverse a given number.
30. What are the rules apply to a #define statement ?
31. Write a program to print the following output using for loop :

```
        1
      2   2
    3   3   3
  4   4   4   4
5   5   5   5   5
```

32. Explain the working of *continue* statement using a suitable example.

33. Explain the handling of multidimensional arrays with syntax and suitable example.

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. (a) Write an algorithm and draw the flowchart of the program to check whether a given number is prime or not. (5 marks)

(b) Explain different storage classes in C. (5 marks)

35. Write an essay on user-defined functions in C.

36. (a) Write a C program to swap to numbers using function. (5 marks)

(b) Write a C program to reverse a string. (5 marks)

[2 × 10 = 20 marks]

C 20212-A (Pages : 4) Name.....................................

Reg. No.................................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

(2014 to 2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 6B 13 (E02)—LINEAR PROGRAMMING

(Multiple Choice Questions for SDE Candidates)

1. For any two points x and y in $\mathbb{R}^n$, the set $\{u : u = \lambda x + (1 - \lambda)y, 0 \le \lambda \le 1\}$ is called :

(A) The circle. (B) A parabola.

(C) An ellipse. (D) The line segment joining the points x and y.

2. The intersection of a finite number of convex sets is :

(A) Not convex. (B) A half space.

(C) A cone. (D) Convex.

3. A set that contains the e-nbd of each of its points, is called :

(A) The boundary of the set. (B) The closure of the set.

(C) An open set. (D) A closed set.

4. A set is said to be closed if its, :

(A) Complement is open. (B) Complement is not open.

(C) Complement is convex. (D) Complement is not convex.

5. Let S be a convex subset of the plane, bounded by lines in the plane. Then a linear function $z = c_1x_1 + c_2x_2$, where $x_1, x_2 \in S$, c_1 and c_2 are scalars, attains its extreme values at :

(A) The interior of S. (B) The vertices of S.

(C) The exterior of S. (D) The X-axis.

6. In the itertion of simplex method, if $z_j - c_j \ge 0$ for all j, then the initial basic feasible solution is :

(A) Not a solution. (B) Not optimal.

(C) An optimum solution. (D) None of the above.

7. A feasible solution to an L.P.P. which is also a basic solution to the problem is called :

(A) An optimum solution to the L.P.P.

(B) A standard solution to the L.P.P.

(C) A basic feasible solution to the L.P.P.

(D) A feasible solution to the L.P.P.

8. If the number of dual variables are m adn primal constraints are n, then :

(A) $m \ne n$. (B) $m > n$.

(C) $m < n$. (D) $m = n$.

9. The dual of dual problem is :

(A) The unsymmetric dual problem. (B) The unsymmetric primal problem.

(C) The dual problem. (D) The primal problem.

10. A balanced transportation problem has :

(A) An optimal solution always. (B) No solution.

(C) No feasible solution. (D) No optimal solution.

11. A system of n linear equations $\mathbf{Ax} = \mathbf{b}$ is called a triangular system if the matrix $\mathbf{A}$ is :

(A) Unit matrix. (B) Zero matrix.

(C) Diagonal matrix. (D) Triangular matrix.

12. An initial basic feasible solution to a T.P. is obtained by :

(A) Method of penalties. (B) Two-phase simplex method.

(C) Row minima method. (D) Big M method.

13. An L.P.P. can be solve using graphical method if it has :

(A) More than four variables. (B) Only two variables.

(C) More than two variables. (D) Three variables.

14. The assignment problem is a special case of transportation problem because :

(A) The number of origins is not equal to number of destinations.

(B) The number of origins is greater than number of destinations.

(C) The number of origins is less than number of destinations.

(D) The number of origins is equal to number of destinations.

15. Cl (2, 7) = ————.

(A) (2, 7). (B) (2, 7].

(C) [2, 7). (D) [2, 7].

16. A simplex in zero dimension is :

(A) A point. (B) A line.

(C) A square. (D) None of the above.

17. A simplex in one dimension is :

(A) A point. (B) A line segment.

(C) A square. (D) None of the above.

18. The optimal solution to an L.P.P. is :

(A) Always infinite. (B) Always finite.

(C) Unique. (D) Either unique or infinite.

19. The simplex method is :

(A) An iterative method. (B) A direct method.

(C) Both direct and iterative method. (D) None of the above.

20. Loop is associated with :

(A) Two-phase simplex method. (B) Big M method.

(C) Assignment problem. (D) Transportation problem.

C 20212 (Pages : 3) Name.................................

Reg. No.................................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define vertex of a convex set.
2. Find the convex hull of $\{x_1, x_2\} \subset \mathbb{R}^2$.
3. Define objective function.
4. While solving an LPP by graphical method, when does one conclude that there exist infinitely many points in the feasible region at which the objective function attains optimum ?
5. Define a slack variable.
6. Write the matrix form of general LPP.
7. What is meant by feasible solution to an LPP ?
8. What is meant by optimal solution to an LPP ?
9. When do you say that a transportation problem is unbalanced ?
10. State the necessary and sufficient condition for the existence of a feasible solution to general transportation problem.
11. Write the number of basic variables of the general transportation problem at any stage of feasible solution.
12. In assignment problem, what is the value of decision variable ?

(12 × 1 = 12 marks)

Section B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Prove that the intersection of two convex sets is a convex set.
14. Write the standard form of LPP.
15. What is the basic principle of linear programming ?

Turn over

16. Does a feasible solution exist for the LPP : Maximize $z = 2x_1 + 10x_2$ subject to $x_1 - x_2 \geq 1, -x_1 + x_2 \geq 2, x_1 \geq 0, x_2 \geq 0$? Give reasons.

17. For linear inequalities, show that the solution set for a group of inequalities is a convex set.

18. Write the dual of the LPP :

    Maximize $f(x) = 3x_1 + 2x_2$ subject to $2x_1 + x_2 \leq 20, x_1 + 3x_2 \leq 20, x_1 \geq 0, x_2 \geq 0$.

19. Define a loop in transportation table.

20. How does a loop in a transportation table related to a basic feasible solution ?

21. Define a triangular basis. Write the role of triangular basis in transportation problem.

22. How do you solve an unbalanced transportation problem ?

23. Write the disadvantage of North-West corner rule.

24. "An assignment problem is a particular case of a transportation problem." Justify.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Show that the set $S = \{(x_1, x_2) : 5x_1 + 2x_2 \geq 10, 2x_1 + 5x_2 \geq 10\}$ is convex.

26. (a) Define convex linear combination of a finite set of vectors.

    (b) Show that the set of all convex linear combinations of a finite number of vectors $u_1, u_2, u_3, \ldots, u_k \in \mathbb{R}^n$ is a convex set.

27. Prove that the set of feasible solutions to an LPP is a convex set.

28. Show that the following system of linear equations has a degenerate solution :

    $2x_1 + x_2 - x_3 = 2, 3x_1 + 2x_2 + x_3 = 3$.

29. Let $x_1 = 2, x_2 = 4$ and $x_3 = 1$ be a feasible solution to the system of equations $2x_1 - x_2 + 2x_3 = 2, \; x_1 + 4x_2 = 18$. Reduce this feasible solution to a basic feasible solution.

30. Use graphical method to solve the following LPP :

    Maximize $z = 6x_1 + x_2$ subject to $2x_1 + x_2 \geq 3, x_2 - x_1 \geq 0, x_1 \geq 0, x_2 \geq 0$.

31. Prove that the dual of the dual is the primal.

32. How do you resolve the problem of degeneracy in transportation problem ?

33. Obtain an initial basic feasible solution to the following transportation problem using North-West Corner rule :

	D_1	D_2	D_3	D_4	Supply
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand	16	18	31	25	

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Use Simplex method to solve the LPP :

Maximize $Z = 4x_1 + 10x_2$ subject to

$$2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90 \text{ and } x_1 \geq 0, x_2 \geq 0.$$

35. Use Vogel's approximation method to obtain an initial basic feasible solution to the transportation problem :

	D_1	D_2	D_3	D_4	Availability
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
Demand	200	225	275	250	

36. A department head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate of the time each man would take to perform each task is given in the matrix below :

Tasks	Men			
	T_1	T_2	T_3	T_4
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How the task should be allocated, one task to a man, so as to minimize the total man-hours ?

(2 × 10 = 20 marks)

C 20211-A (Pages : 4) Name.................................

Reg. No.................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 13 (E01)—GRAPH THEORY

(2014 to 2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 6B 13 (E01)—GRAPH THEORY

(Multiple Choice Questions for SDE Candidates)

1. Let T be a tree. Then which of the following statements are true :

   (A) There exists two paths between every pair of vertices.

   (B) T is minimally connected.

   (C) T has n vertices and $n-2$ edges.

   (D) T has n vertices and n edges.

2. Let T = (V, E) be a tree. Then the eccentricity of the vertex is given by :

   (A) $e(v) = \max\{d(u,v) : u \in V, u \neq v\}$.

   (B) $e(v) = \min\{d(u,v) : u \in V, u \neq v\}$.

   (C) $e(v) = \min\{d(u) : u \in V\}$.

   (D) $e(v) = \max\{d(u) : u \in V\}$.

3. Let G = (v, E) be a connected graph and let $v \in V$. Then v is a central point if :

   (A) $e(v) > r(G)$.

   (B) $e(v) < r(G)$.

   (C) $e(v) = r(G)$.

   (D) $e(v) \geq r(G)$.

4. Let G = (v, E) be a graph such that $|E| = 8$ and deg $(v) = 2$ for all $v \in V$. Then $|V| =$.

   (A) 6.

   (B) 2.

   (C) 8.

   (D) 10.

5. Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph. Then $G_1 + G_2$ is a :

   (A) $(p_1 + p_2, q_1 + q_2)$ graph.

   (B) $(p_1 q_1, p_2 q_2)$ graph.

   (C) $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$ graph.

   (D) $(p_1 - p_2, q_1 - q_2)$ graph.

6. For what values of n is the graph K_n Eulerian ?

   (A) n is a even number.

   (B) n is an odd number.

   (C) n is a composite number.

   (D) None of these.

7. $K_{m,n}$ is Eulerian if and only if :

   (A) m and n are even.

   (B) m and n are odd.

   (C) m is even and n is odd.

   (D) m is odd and n is even.

8. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. Let f be an isomorphism from G_1 to G_2. Let $v \in V_1$. Then which of the following statements are true ?

(A) $\deg(v) = \deg(f(v))$. (B) $\deg(v) > \deg(f(v))$.

(C) $\deg(v) < \deg(f(v))$. (D) $\deg(v) \neq \deg(f(v))$.

9. The number of perfect matchings in the complete bipartite graph $K_{m,n}$ is :

(A) n. (B) $n!$.

(C) $(n-1)!$ (D) n^2.

10. The number of perfect matching in K_{2n} is :

(A) $2n!$. (B) $\frac{2n!}{n!}$.

(C) $\frac{2n!}{2^n}$. (D) $\frac{2n!}{2^n n!}$.

11. Let G be a (p, q) plane graph in which every face is an n-cycle then $q =$

(A) $\frac{n(p-2)}{n-2}$. (B) $\frac{(p-2)}{n-2}$.

(C) $\frac{n(p+2)}{n-2}$. (D) $\frac{n(p-2)}{n+2}$.

12. Which of the following pairs of graphs are not planar ?

(A) K_2, $K_{2,2}$. (B) K_3, $K_{2,2}$.

(C) K_4, $K_{2,2}$. (D) K_5, $K_{3,3}$.

13. If a (p, q) graph is self dual, the $2p - q =$

(A) 1. (B) 0.

(C) 2. (D) 4.

14. Every block of a tree is :

(A) K_2. (B) K_3.

(C) K_4. (D) K_5.

15. Which of the following statements are true :

(A) Every Hamiltonian graph is 0-connected.

(B) Every Hamiltonian graph is 1-connected.

(C) Every Hamiltonian graph is 2-connected.

(D) Every Hamiltonian graph is 3-connected.

16. The value of r (2, 2) =

(A) 0. (B) 1.

(C) 2. (D) 3.

17. The graph $K_{r,s}$ is Hamiltonian if :

(A) $r > s$. (B) $r < s$.

(C) $r \neq s$. (D) $r = s$.

18. The chromatic number of $K_{m,n}$ is :

(A) 1. (B) 2.

(C) 3. (D) 4.

19. The chromatic number of C_{2n} is :

(A) 1. (B) 2.

(C) 3. (D) 4.

20. The value of r (3, 3) is :

(A) 0. (B) 1.

(C) 4. (D) 6.

C 20211 (Pages : 3) Name.....................................

Reg. No..................................

SIXTH SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 13 (E01)—GRAPH THEORY

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. An edge joining a vertex to itself is called a ————.
2. Define simple graph.
3. Define complete graph.
4. Define Vertex degree.
5. Draw a 3-regular graph.
6. Define walk.
7. Define a tree.
8. Define bridge of a graph.
9. What is the number of different spanning trees of the complete graph K_n ?
10. Define cut vertex of a graph.
11. Define Hamiltonian cycle.
12. State Euler's formula for a plane graph.

(12 × 1 = 12 marks)

Section B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

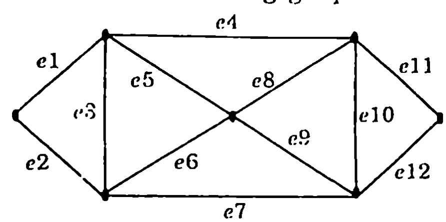
13. Define Graph.
14. For any graph with n vertices $v_1, v_2, \ldots v_n$ and ε edges, show that

$$\sum_{i=1}^{n} d(v_i) = 2\epsilon .$$

15. Define complement of a graph. Give an example.

Turn over

16. Let G be a graph with n vertices and exactly $n - 1$ edges. Prove that G has either a vertex of degree 1 or an isolated vertex.
17. Draw the Petersen graph.
18. If G is an acyclic graph with k connected components, then show that G has $n - k$ edges.
19. If every edge of a graph G is a bridge, then show that G is a forest.
20. If G is an acyclic graph with $n - 1$ edges, then show that G is connected.
21. If v is a cut vertex of a graph G, then show that there are two vertices u and w of G, both different from v, such that v is on every $u - w$ path in G.
22. Give an Euler tour for the following graph :



23. Show that the dodecahedron is Hamiltonian.
24. Give an example for a planar graph and a non-planar graph.

(9 × 2 = 18 marks)

Section C

Answer any **six** *question.*
Each question carries 5 *marks.*

25. In any graph G, show that there is an even number of odd vertices.
26. Given any two vertices u and v of a graph G, show that every $u - v$ walk contains a $u - v$ path.
27. Draw the graph whose adjacency matrix is given by $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$.
28. Let G be a graph with n vertices $v_1, v_2, \ldots, v_n$ and let A denote the adjacency matrix of G with respect to this listing of the vertices. Let $B = [b_{i,j}]$ be the matrix $B = A + A^2 + \ldots + A^{n-1}$. Show that G is connected if and only if B has no zero entries off the main diagonal.
29. Let T be a tree with at least two vertices and let $P = u_0 u_1 \ldots u_n$ be a longest path in T. Then show that both u_0 and u_n have degree 1.
30. If T is a tree with n vertices then show that it has precisely $n - 1$ edges.

31. Write a note on Kongsberg bridge problem.

32. Prove that it is impossible to have a group of nine people at a party such that each one known exactly five of the others in the group.

33. Let e be an edge of a connected graph G. Show that e is a bridge if and only if it is in every spanning tree of G.

(6 × 5 = 30 marks)

Section D

Answer any **two** *question.*
Each question carries 10 *marks.*

34. Let G be a non-empty graph with at least two vertices. Then show that G is bipartite if and only if it has no odd cycles.

35. (a) Show that a graph G is connected if and only if it has a spanning tree.

    (b) Give an example of a non Hamiltonian connected graph.

36. (a) Let G be a graph in which the degree of every vertex is at least two, then show that G is acyclic.

    (b) Prove that the wheel W_n is Hamiltonian for every $n \geq 4$.

(2 × 10 = 20 marks)

C 20210-A (Pages : 5) Name.................................

Reg. No..............................

SIXTH SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

(2014 to 2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 30**	**Maximum : 30 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 30.
2. The candidate should check that the question paper supplied to him/her contains all the 30 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

(Multiple Choice Questions for SDE Candidates)

1. A subset W of a vector space V is a subspace if :
   - (A) The sum of elements of W belongs to W.
   - (B) The sum and scalar multiples of elements of W belongs to W.
   - (C) The scalar multiples of elements of W belongs to W.
   - (D) The sum of elements of V belongs to W.
2. The intersection of any set of subspaces of a vector space V is :
   - (A) Not a subspace of V.
   - (B) A subspace of V.
   - (C) Need not be a subspace of V.
   - (D) A proper subspace of V.
3. The union of any two subspaces of a vector space V is :
   - (A) A subspace of V.
   - (B) Not a subspace of V.
   - (C) Need not be a subspace of V.
   - (D) A proper subspace of V.
4. If a vector space V is of dimension n then every subset containing more than n elements is :
   - (A) Linearly independent.
   - (B) Linearly dependent.
   - (C) Neither linearly independent nor dependent.
   - (D) A basis.
5. If W is a subspace of a finite dimensional vector space V then :
   - (A) $\dim V = \dim W$.
   - (B) $\dim V \leq \dim W$.
   - (C) $\dim V \geq \dim W$.
   - (D) $\dim V \neq \dim W$.
6. The subset $\{(1, 1), (1, -1)\}$ of real vector space $\mathbb{R}^2$ is :
   - (A) Linearly independent only.
   - (B) Linearly dependent.
   - (C) Neither linearly independent nor dependent.
   - (D) A basis.

7. A subset of a linearly independent set is :

(A) Linearly independent.

(B) Linearly dependent.

(C) Neither linearly independent nor dependent.

(D) Need not be linearly independent.

8. If the map $f : V \to W$ is linear. Then the kernel or null space of f is :

(A) $f^{\to}(W)$.

(B) $f^{\to}(\{0_V\})$.

(C) $f^{\leftarrow}(\{0_W\})$.

(D) $f^{\leftarrow}(W)$.

9. Suppose a and b are integers with $a \neq 0$, then $a \mid b$ if :

(A) $a = bc$, c is some integer.

(B) $b = ac$, c is some integer.

(C) $c = ab$, c is some integer.

(D) $ab = 1$.

10. If a and b are two non-zero integers and $gcd(a, b) = d$, then d is the greatest positive integer with :

(A) $d \mid a$ and $d \mid b$.

(B) $d \mid a$ only.

(C) $d \mid b$ only.

(D) $a \mid d$ and $b \mid d$.

11. $gcd(-5, 5) =$ ———.

(A) 3.

(B) 1.

(C) 5.

(D) – 5.

12. $gcd(-8, 36) =$ ———.

(A) 36.

(B) – 8.

(C) 8.

(D) 4.

13. Let a and b be integers, not both zero. Then a and b are relatively prime if and only if there exists integers x and y such that :

(A) $1 = ax + by$.

(B) $2 = ax + by$.

(C) $ab = ax + by$.

(D) $a - b = ax + by$.

14. If a and b are non-zero integers with $a \mid b$, then $gcd(a, b) =$ ———.

(A) $|a|$.

(B) b.

(C) ab.

(D) a.

Turn over

15. Two integers a and b, not both of which are zero, are said to be relatively prime if :

(A) $gcd\,(a, b) = a$.
(B) $a \mid b$.
(C) $gcd\,(a, b) = 1$.
(D) $b \mid a$.

16. If a, b, c are any integers with $a \mid bc$ and $gcd(a, b) = 1$, then :

(A) $b \mid a$.
(B) $a \mid c$.
(C) $c \mid a$.
(D) $c \mid b$.

17. If a is an odd integer then $gcd(3a, 3a + 2) =$ ————.

(A) 3.
(B) 5.
(C) 1.
(D) 2.

18. Every positive integer $n > 1$ can be expressed as a product of :

(A) Composite numbers.
(B) Prime numbers.
(C) Even numbers.
(D) Odd numbers.

19. If p is a prime and $p \mid ab$, then :

(A) $p \mid a$ only.
(B) $p \mid b$ only.
(C) $p \mid a$ or $p \mid b$.
(D) $p \mid a$ and $p \mid b$.

20. The *Sieve of Eratosthenes* is used for finding :

(A) All primes below a given integer.
(B) All even numbers below a given integer.
(C) All odd numbers below a given integer.
(D) All composite numbers below a given integer.

21. The International Standard Book Number (ISBN) consists of nine digits $a_1a_2,\ldots a_9$ followed by a tenth check digit a_{10}, which satisfies :

(A) $a_{10} \equiv \sum_{k=1}^{9} ka_k \pmod 9$.
(B) $a_{10} \equiv \sum_{k=1}^{9} ka_k \pmod{11}$.
(C) $a_{10} \equiv \sum_{k=1}^{9} ka_k \pmod{10}$.
(D) $a_{10} \equiv \sum_{k=1}^{9} ka_k \pmod 7$.

22. Any *palindrome* with even number of digits is divisible by :

(A) 5.
(B) 2.
(C) 11.
(D) 12.

23. The number of primes of the form $n^2 - 2$ is :

(A) Finite. (B) Infinite.

(C) 1729. (D) 1.

24. If p and q are distinct prime with $a^p \equiv a(\text{mod } q)$ and $a^q \equiv a(\text{mod } p)$, then :

(A) $a^{p+q} \equiv a(\text{mod } pq)$. (B) $a^{pq} \equiv a(\text{mod } pq)$.

(C) $a^{p-q} \equiv a(\text{mod } pq)$. (D) $a^{p/q} \equiv a(\text{mod } pq)$.

25. A composite number n for which $a^n \equiv a\,(\text{mod } n)$ is called :

(A) A pseudoprime. (B) A prime.

(C) A pseudoprime to the base a. (D) An absolute pseudoprime.

26. If p is a prime, then $(p-1)! + 1$ is :

(A) a multiple of p. (B) a multiple of $p - 1$.

(C) a multiple of $p + 1$. (D) a multiple of p^2.

27. If n and r are positive integers with $1 \leq r < n$, then the binomial coefficient $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ is :

(A) An integer. (B) A prime.

(C) Irrational. (D) An even integer.

28. For $n > 2$, $\phi(n)$ is :

(A) An odd integer. (B) An even integer.

(C) Irrational. (D) Prime.

29. $\phi(360) =$ ————.

(A) 96. (B) 98.

(C) 86. (D) 90.

30. For any integer a, $a^{37} \equiv$ ———— (*mod* 1729).

(A) a. (B) $2a$.

(C) a^2. (D) $a - 1$.

C 20210 (Pages : 3) Name..................................

Reg. No..................................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. State general version of the Division Algorithm.
2. Find lcm (306, 657).
3. State the Fundamental Theorem of Arithmetic.
4. Explain the method of Sieve of Eratosthenes.
5. Define Pseudoprime. Give an example of a Pseudoprime number.
6. Find $\sigma(12)$.
7. Define Euler's Phi Function.
8. Prove that the set $X = \{(x, 0) : x \in \mathbb{R}\}$ is a subspace of the vector space $\mathbb{R}^2$.
9. Define basis of a vector space.
10. Check whether the map $f : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $f(x,y,z) = (x+y,z,0)$ is linear. Justify your claim.
11. Define the rank and nullity of the linear map.
12. For $(x,y) \in \mathbb{R}^2$, let $S = \{(x,y)\}$. Find $\langle S \rangle$.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Show that square of any odd integer is of the form $8k + 1$.
14. If $a \mid c$ and $b \mid c$, with $gcd(a, b) = 1$, prove that $ab \mid c$.
15. Use the Euclidean Algorithm to obtain integers x and y satisfying $gcd(24, 138) = 24x + 138y$.
16. Determine all solutions in the positive integers of the Diophantine equation $18x + 5y = 48$.

Turn over

17. Prove that the number $\sqrt{2}$ is irrational.

18. For arbitrary integers a and b, prove that $a \equiv b (\text{mod } n)$ if and only if a and b leave the same non-negative remainder when divided by n.

19. Use the binary exponential algorithm to compute $5^{110} (\text{mod } 131)$.

20. Solve the linear congruence $18x \equiv (\text{mod } 42)$.

21. If p and q are distinct primes with $a^{p} \equiv a (\text{mod } q)$ and $a^{q} \equiv a (\text{mod } p)$, then prove that $a^{pq} \equiv a (\text{mod } pq)$.

22. Prove that intersection of two subspaces of vector space V over a field is again a subspace.

23. Let A = Span {(1, 2, 0, 1), (– 1, 1, 1, 1)} and B = Span {(0, 0, 1, 1), (2, 2, 2, 2)} be two subspaces of $\mathbb{R}^4$. Determine $A \cap B$ and compute its dimension.

24. If V is a vector space over the set $\mathbb{C}$ of complex numbers of dimension n, prove that V can be regarded as a vector space over $\mathbb{R}$ of dimension $2n$.

25. Let $f : V \to W$ be linear. Prove that if Y is subspace of W then $f^{\leftarrow}(Y)$ is a subspace of V.

26. Let V be a vector space of dimension $n \geq 1$ over a field F. Prove that V is isomorphic to the vector space F^n.

(10 × 4 = 40 marks)

Section C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Given integers a and b, not both of which are zero, prove that there exists integers x and y such that $gcd\ (a, b) = ax + by$.

28. Prove that the linear Diophantine equation $ax + by = c$ has an integer solution if and only if $d \mid c$ where $d = \gcd(a, b)$.

29. Prove that there are infinite number of primes.

30. Using Chinese Remainder Theorem, solve the system of congruences $x \equiv 5 (\text{mod } 11), x \equiv 14 (\text{mod } 29), x \equiv 15 (\text{mod } 31)$.

31. If p is a prime, then prove that $(p-1)! \equiv -1 (\text{mod } p)$.

32. Prove that a non-empty subset S of a vector space V is a basis of V if and only if every element of V can be expressed in a unique way as a linear combination of elements of S.

33. Let $f, g, h : \mathbb{R} \to \mathbb{R}$ be the mappings given by $f(x) = \cos^2 x, g(x) = \sin^2 x, h(x) = \cos 2x$. Consider the subspace of $\text{Diff}(\mathbb{R}, \mathbb{R})$ given by W = Span $\{f, g, h\}$. Find a basis for W.

34. Show that the linear mapping $f:\mathbb{R}^3 \to \mathbb{R}^3$ given by $f(x,y,z)=(x+y+z,\, 2x-y-z, x+2y-z)$ is both surjective and injective.

35. State and prove Dimension Theorem.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. (a) If $a = qb + r$, prove that $\gcd(a,b) = \gcd(b,r)$. (4 marks)

(b) If a cock is worth 5 coins, a hen 3 coins, and three chicks together 1 coin, how many cocks, hens, and chicks, totalling 100, can be bought for 100 coins.

(9 marks)

37. Prove that the lienar congruence $ax \equiv b (\bmod n)$ has a solution if and only if $d \mid b$, where $d = \gcd(a,n)$. Moreover if $d \mid b$, prove that it has d mutually incongruent solutions modulo n.

38. Prove that every linearly independent subset I of a finite dimensional vector space V can be extended to form a basis.

[2 × 13 = 26 marks]

C 20209 (Pages : 4) Name.....................................

Reg. No.................................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 11—NUMERICAL METHODS

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What is the minimum number of iterations required in bisection method to achieve an accuracy ϵ ?
2. State the condition for convergence of Newton-Raphson method.
3. Define the central difference operator.
4. Evaluate $\Delta\left(x^2 + \sin x\right)$, interval of differencing being h.
5. State Newton's backward difference interpolation formula.
6. Show that the Lagrange interpolating polynomial is unique.
7. Given $f(x) = \dfrac{1}{x^2}$, find the divided differences $[a, b]$ and $[a, b, c]$.
8. Given a set of n-values of (x, y), what is the formula for computing $\left[\dfrac{d^2y}{dx^2}\right]_{x_n}$.
9. State general formula for numerical integration.
10. What is complete pivoting ?
11. Write Runge-Kutta formula to fourth order to solve $\dfrac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.
12. Write Adams-Moulton corrector formula.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Given that the equation $x^{2.2} = 69$ has a root between 5 and 8. Use the methods of Regula-Falsi to determine it.

Turn over

14. Prove that (i) $\delta = \Delta E^{1/2}$; (ii) $E = e^{hD}$ where E is the shift operator and D is the differential operator.

15. Given $\log_{10} 100 = 2$, $\log_{10} 101 = 2.0043$, $\log_{10} 103 = 2.0128$, $\log_{10} 104 = 2.0170$, find $\log_{10} 102$.

16. The function $y = \sin x$ is tabulated below :

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y = \sin x$	0	0.70711	1.0

Using Lagrange's interpolation formula, find the value of $\sin\left(\frac{\pi}{6}\right)$.

17. Prove that the nth divided difference of a polynomial of nth degree are constant.

18. Given the set of tabulated points (0, 2), (1, 3), (2, 12) and (15, 3587) satisfying the function $y = f(x)$, compute $f(4)$ using Newton's divided difference formula.

19. Using Simpson's $\frac{3}{8}$-rule with $h = \frac{\pi}{6}$, evaluate the integral $\int_0^{\pi/2} \sin x \, dx$.

20. Solve the system $2x + y + z = 10; 3x + 2y + 3z = 18; x + 4y + 9z = 16$ by the Gauss-Jordan method.

21. Decompose the matrix $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ in the form LU where L is a unit lower triangular matrix and U is a upper triangular matrix.

22. Find the smallest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

23. Use Picard's method to obtain $y(0.1)$ of the problem defined by $\frac{dy}{dx} = x + yx^4, y(0) = 3$.

24. Explain briefly the method of iteration to compute a real root of the equation $f(x) = 0$, stating the condition of convergence of the sequence of approximations.

25. A rod is rotating in a plane about one of its ends. The angle θ (in radians) at different times t (seconds) are given below :

t	0	0.2	0.4	0.6	0.8	1.0
θ	0.0	0.15	0.50	1.15	2.0	3.20

Find its angular acceleration when $t = 0.6$ seconds.

26. Solve the tridiagonal system of equations $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 2 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Using the secant method, find a real root of the equation $f(x) = xe^x - 1 = 0$.

28. Using bisection method find the positive root, between 0 and 1, of the equation $x = e^{-x}$ to a tolerance of 0.05 %.

29. Using Newton's forward interpolation formula, find y at $x = 8$ from the following table :

x	0	5	10	15	20	25
y	7	11	14	18	24	32

30. From the following table, find the value of $e^{1.17}$ using Gauss' forward formula :

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

31. Given the table of values

x	2	3	4	5
x^3	8	27	64	125

Use the method of successive approximations to find x when $x^3 = 10$.

32. Find the first and second derivatives of the function tabulated below at the point $x = 2.2$:

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

33. Use Gauss elimination to find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{bmatrix}$.

34. If $\dfrac{dy}{dx} = \dfrac{1}{x^2 + y}$ with $y(4) = 4$ compute the values of $y(4.1)$ and $y(4.2)$ by Taylor's series method.

35. A curve is given by the points of the table given below :

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	23	19	14	11	12.5	16	19	20	20

Apply Simpson's rule to find the area bounded by the curve, the x-axis and the extreme ordinates.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. Evaluate $\int_0^1 \frac{dx}{1+x}$ using :

(a) Trapezoidal rule taking $h = 0.25$.

(b) Simpson's $\frac{1}{3}$-rule taking $h = 0.125$.

37. Solve the system $10x + 2y + z = 9; 2x + 20y - 2z = -44; -2x + 3y + 10z = 22$ using both Jacobi and Gauss-Seidel method.

38. (a) Use Runge-Kutta fourth order formula to find $y(0.2)$ and $y(0.4)$ given that $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1$.

(b) Solve the initial value problem $\frac{dy}{dx} = 1 + y^2, y(0) = 0$ with $h = 0.2$ on the interval [0, 0.6] using Milne's method.

(2 × 13 = 26 marks)

C 20208-A (Pages : 6) Name..................................

Reg. No..................................

SIXTH SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

(2014 to 2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 30**	**Maximum : 30 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 30.
2. The candidate should check that the question paper supplied to him/her contains all the 30 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 6B 10—COMPLEX ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. Real part of $f(z) = \log z$ is :

   (A) $\frac{1}{2}\log\left(x^2 + y^2\right)$. (B) $\log\left(x^2 + y^2\right)$.

   (C) $\log(x + iy)$. (D) None of these.

2. If n is a positive integer, then $\left(1 + i\sqrt{3}\right)^n + \left(1 - i\sqrt{3}\right)^n$ is equal to :

   (A) $2^n \sin\frac{n\pi}{2}$. (B) $2^{n+1} \cos\frac{n\pi}{3}$.

   (C) $2^{n+1} \sin\frac{n\pi}{3}$. (D) None of these.

3. Real part of $f(z) = z^3$ is :

   (A) $x^3 - 3xy^2$. (B) $x^3 + 3xy^2$.

   (C) $x^3 - 3x^2y$. (D) None of these.

4. Value of $(1 + i)^{24}$ is :

   (A) 2^{24}. (B) $\left(\sqrt{2}\right)^{24} e^{\frac{i\pi}{4}}$.

   (C) 2^{12}. (D) None of these.

5. If $f(z)$ is a real valued analytic function in a domain D, then :

   (A) $f(z)$ is a constant. (B) $f(z)$ is identically zero.

   (C) $f(z)$ has modulus 1. (D) None of these.

6. The function e^{iz} has period :

   (A) $2\pi i$. (B) 2π.

   (C) π. (D) 3π.

7. For real numbers x and y, $\sin(x + iy)$ equals :

   (A) $\sin x \cosh y + i \cos x \sinh y$. (B) $\cos x \cosh y - i \sin x \sinh y$.

   (C) $\sin x \cosh y - i \cos x \sinh y$. (D) $\cos x \cosh y + i \sin x \sinh y$.

8. Real part of the function $f(z) = |z|^2$ equals :

(A) $2xy$.
(B) $x^2 - y^2$.
(C) $x^2 + y^2$.
(D) None of these.

9. Which of the following is not a simply connected region ?

(A) Circular disk.
(B) Half planes.
(C) An annulus region.
(D) A parallel strip.

10. The integral $\int_{|z|=2\pi} \frac{\sin z}{(z-\pi)^2} dz$ where the curve is taken anti-clockwise, equals :

(A) $-2\pi i$.
(B) $2\pi i$.
(C) 0.
(D) $4\pi i$.

11. The value of the integral $\int_C \frac{dz}{(z-a)^{10}}$, where C is $|z - a| = 3$ is :

(A) 0.
(B) $2\pi i$.
(C) πi.
(D) None of these.

12. The value of the integral $\int_C \frac{e^{5z}}{z^3} dz$, where C is $|z| = 3$ is :

(A) $10\pi i$.
(B) $2\pi i$.
(C) $25\pi i$.
(D) None of these.

13. Value of the integral $\int_0^x e^{it} dt$ is :

(A) $2i$.
(B) 0.
(C) $2\pi i$.
(D) None of these.

14. If n is any non-zero integer, then $\int_0^{2\pi} e^{in\theta} d\theta$ equals :

(A) 0.
(B) 2π.
(C) 1.
(D) None of these.

15. Converse of Cauchy's integral theorem is known as :

(A) Liouville's theorem.
(B) Goursat's theorem.
(C) Morera's theorem.
(D) Euler's theorem.

16. A Maclaurin series is a Taylor series with centre :

(A) $z_0 = 1$.

(B) $z_0 = 0$.

(C) $z_0 = 2$.

(D) None of these.

17. The radius of convergence of the power series of the function $f(z) = \frac{1}{1-z}$ about $z = 1/4$ is :

(A) 1.

(B) ¼.

(C) ¾.

(D) 0.

18. If $f(z)$ is entire, then $f(z) = \sum_{n=1}^{\infty} a_n z^n$ has radius of convergence :

(A) 0.

(B) e.

(C) ∞.

(D) None of these.

19. A power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ always converges for :

(A) at least one point z.

(B) all complex numbers z.

(C) at all z which are either real or purely imaginary.

(D) at all z with $|z - z_0| < R$ for some $R > 0$.

20. A function $f(z)$ given by a power series is analytic at :

(A) Every point of its domain.

(B) Every point inside its circle of convergence.

(C) Every point on the circle of convergence.

(D) Every point in the complex plane.

21. The singular points of the function $f(z) = \frac{1}{4z - z^2}$ are :

(A) $z = 0$ and $z = -4$.

(B) $z = 0$ and $z = 4$.

(C) $z = 4$ and $z = -4$.

(D) $z = 2$ and $z = -2$.

22. The constant term in the Laurent series expansion of $f(z) = \frac{e^z}{z^2}$ in the region $0 < |z| < \infty$ is :

(A) 0.
(B) ½.
(C) 2.
(D) None of these.

23. If $f(z)$ has a zero of order m at z_0 and $g(z)$ has a pole of order n at z_0 and $n \le m$, then the product $f(z)g(z)$ has at z_0 :

(A) An essential singularity.
(B) A pole of order $m - n$.
(C) A removable singularity.
(D) A pole of order $m - 1$.

24. If $f(z)$ has a pole of order m at z_0, then $g(z) = \frac{f'(z)}{f(z)}$, at z_0 has :

(A) A simple pole.
(B) A pole of order m.
(C) A pole of order $m + 1$.
(D) A pole of order $m - 1$.

25. For $f(z) = \frac{\tan z}{z}$, $z = 0$ is a :

(A) Essential singularity.
(B) Simple pole.
(C) Removable singularity.
(D) Double pole.

26. Which of the following function has a simple zero at $z = 0$ and an essential singularity $z = 1$?

(A) $ze^{\frac{1}{z-1}}$.
(B) $ze^{\frac{1}{1+z}}$.
(C) $(z-1)e^{\frac{1}{z}}$.
(D) $(z-1)e^{\frac{1}{z-1}}$.

27. The function $f(z) = \frac{z^2 + 2iz + 3}{(z-i)^2(z+i)}$ at $z = i$ has :

(A) Regular point.
(B) Simple pole.
(C) Double pole.
(D) Removable singularity.

28. Singularities of a rational function are :

(A) Poles.
(B) Essential.
(C) Non-isolated.
(D) Removable.

29. The singularity of the function $\frac{\sin z}{z}$ at $z = 0$ is :

(A) Essential singularity.

(B) Simple pole.

(C) Removable singularity.

(D) Double pole.

30. At $z = 1$, the residue of $f(z) = \frac{z^2 + 1}{z(z-1)}$ is :

(A) – 1.

(B) 0.

(C) 1.

(D) 2.

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SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. A complex function $f(z)$ is analytic at a point $z = z_0$ if ———.
2. An analytic function with constant argument is ———.
3. Give an example of a complex function which is Differentiable at a point but not analytic at that point.
4. Find the simple poles, if any for the function $f(z) = \dfrac{(z+2)^2}{z^5(x^4-1)}$.
5. Write the polar form of Cauchy-Riemann equations.
6. Define residue of a complex valued function.
7. Fill in the blanks : The real part of sinh (2z) is ———.
8. Fill in the blanks : $f(z) = e^z$ is periodic with period = ———.
9. A point $z = z_0$ is a singular point of a complex function $w = f(z)$ if ———.
10. Fill in the blanks : $\text{Res}_{z=\pi/2} \tan z =$ ———.
11. The solution of the equation $e^z = -3$ is ———.
12. The principal value of i^i is ———.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Show that $f(z) = \sin z$ is analytic for all z.
14. Find the principal value of $(1 - i)^{1+i}$.
15. Show that $\tanh^{-1}(z) = \dfrac{1}{2}\log\dfrac{1+z}{1-z}$.
16. Show that the zeros of an analytic function are isolated.

Turn over

17. Determine and classify the singular points of $f(z) = \frac{(z+2)^2}{z^5(z^4-1)}$.

18. Find the radius of convergence of the power series : $\sum_{n=0}^{\infty} \frac{n!z^n}{n^n}$.

19. Verify Cauchy-Groursat theorem for $f(z) = z^5$ when the contour of integration is the circle with centre at origin and radius 3 units.

20. Discuss the nature of singularities if any, of $f(z) = \sin(1/z)$ in the complex plane.

21. Find all the solution of $e^z = 2$.

22. Find the residue of $f(z) = \cot(z)$ at its poles.

23. Evaluate $\oint_C \frac{\sin \pi z}{(z^6)} dz$ around $C = |z| = 1$.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Evaluate $\oint_{|z|=2} \bar{z} dz$.

26. Illustrate entire function by an example.

(10 × 4 = 40 marks)

Section C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Evaluate $\oint_C \frac{1}{(z-1)(z-2)}$ around the simple closed curve $C = |z| = 4$.

28. Determine the nature of the singularities of the function $f(z) = \sec(1/z)$.

29. Expand $f(z) = \frac{1}{(z+1)(z+2)}$ as a Laurent series valid for $0 < |z+1| < 2$.

30. If $f(z) = u(x, y + iv(x, y))$ is analytic in a domain D, then prove that its component functions are harmonic in D.

31. Find the analytic function $f(z)$ is terms of z, if $u(x, y) = \text{Re}(f(z)) = e^x(x\cos y - y\sin y)$.

32. Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin, even though Cauchy Riemann equations are satisfied at that point.

33. State and prove Morera's theorem.

34. Show that the derived series has the same radius of convergence as the original series.

35. Evaluate $\oint_{|z-2|=2} \frac{z^3}{(z-1)^4(z-2)(z-3)} dz$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. (a) State and prove Cauchy's integral formula.

    (b) Prove or disprove : $|\cos(z)| \leq 1$ for all complex numbers z. Justify your claim.

37. (a) State and prove fundamental theorem of Algebra.

    (b) Find the residues of $f(z) = \dfrac{z^2}{(z-1)^2(z-2)}$ at its poles.

38. (a) Evaluate using the method of residues : $\int_0^{2\pi} \dfrac{1}{a+\cos\theta} d\theta$.

    (b) Evaluate $\int_0^{\infty} \dfrac{1}{x^4+a^4} dx, a > 0$.

(2 × 13 = 26 marks)

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SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 09—REAL ANALYSIS

(2014 to 2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 30**	**Maximum : 30 Marks**

INSTRUCTIONS TO THE CANDIDATE

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MAT 6B 09—REAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. Let A be the open interval (– 1, 1). Then, the cluster points of A are :

   (A) 1 and – 1 only.

   (B) All the points of (– 1, 1).

   (C) All the points of [– 1, 1].

2. Which of the following have no cluster points ?

   (A) (3, 4].

   (B) [2, 3].

   (C) {2, 3, 4}.

3. Which among these functions are continuous on $A = \{x \in \mathbb{R}, x \geq 0\}$?

   (A) $f(x) = \sin x$.

   (B) $f(x) = \sin\left(\frac{1}{x}\right)$.

   (C) $f(x) = \frac{1}{x}$.

4. Let $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$ Then $\underset{x \to 0}{\text{Lt}} f(x)$ is :

   (A) 1.

   (B) – 1.

   (C) None of these.

5. Given a, b satisfy $0 < a < 1, b > 1$. Then which of the following sequences is convergent :

   (A) $\frac{ab^n}{2^n}$.

   (B) $\frac{2^{3n}}{3^{2n}}$.

   (C) $\frac{n}{b^n}$.

6. Function $f(x)=\dfrac{1}{2x}$ defined on $A=(0,\infty)$ is an example for :

(A) A continuous function that is not bounded.

(B) A bounded function that is continuous.

(C) An unbounded function that is not continuous.

7. Let $f : A \to \mathbb{R}$ and let L be a real number such that for every $\epsilon > 0$, there exist a $\delta(\epsilon) > 0$ such that if $x \in A$ and $0 < |x - c| < \delta(\epsilon)$, then $|f(x) - L| < \epsilon$. Then L is a ———— of 'f' at c.

(A) Derivative.

(B) Limit.

(C) None of these.

8. When the graph of a function 'f' in $\mathbb{R}$ has a break such that the function value and the limit (does not exist) aren't the same, then 'f' is said to have a ———— discontinuity at that point.

(A) Removable.

(B) Jump.

(C) None of these.

9. When the graph of a function 'f' in $\mathbb{R}$ has a hole in the graph such that the function value and the limit aren't the same, then 'f' has a ———— discontinuity at that point.

(A) Removable.

(B) Jump.

(C) None of these.

10. Let $A \subseteq \mathbb{R}$ and let $f : A \to \mathbb{R}$. If there exist a point $x' \in A$ such that $f(x') \geq f(x)$, for all $x \in A$, then 'f' has an ———— on A.

(A) Absolute maximum.

(B) Absolute minimum.

(C) None of these.

11. Let $f : I \to \mathbb{R}$ be continuous. Then 'f' has an absolute maximum and absolute minimum on I if :

(A) I is closed.

(B) I is closed and bounded not bounded.

(C) I is bounded but not closed.

12. Consider $f(x) = 6x^3 - 3x^2 + 2$. Then which of the following is true ?

    (A) 'f' has a zero in (0, 1).

    (B) 'f' has no zero.

    (C) 'f' has a zero in (– 1, 1).

13. Which of the following statements is true ?

    (A) Every continuous function is uniformly continuous.

    (B) Every uniformly continuous function is continuous.

    (C) None of these.

14. Suppose f and g are in R $[a, b]$ such that $f(x) \leq g(x)$ for all $x \in [a, b]$, then :

    (A) $\int_a^b f(x) < \int_a^b g(x)$.

    (B) $\int_a^b f(x) \leq \int_a^b g(x)$.

    (C) None of these.

15. Which of the following is true ?

    (A) $f \in R[a, b] \Rightarrow f$ is bounded.

    (B) $f \in R[a, b] \Rightarrow f$ is continuous.

    (C) $f \in R[a, b] \Rightarrow f$ is monotone.

16. Let I = [2, 9] which of the following partitions of I has the largest norm ?

    (A) $P_1 = (2, 4, 6, 8, 9)$.

    (B) $P_2 = (2, 6, 9)$.

    (C) $P_3 = (2, 5, 7, 9)$.

17. Let $x \in [0, 1]$ and 'f' be such that $f(x) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational.} \end{cases}$ Then :

    (A) $f \in R[0,1]$.

    (B) $f \notin R[0,1]$.

    (C) None of these.

18. Suppose $f : [a,b] \to \mathbb{R}$ and $f(x) = 0$ except for finite number of points $c_1, c_2, \cdots c_n$ in $[a, b]$. Then :

(A) $f \notin R[a,b]$.

(B) $\int_a^b f = 0$.

(C) None of these.

19. Let $f \in R[a,b]$ and $c \in \mathbb{R}$. Define g on $[a+c, b+c]$ by $g(y) = f(y-c)$. Then which of these holds ?

(A) $\int_a^b f = \int_a^b g$.

(B) $\int_a^b f = \int_{a+c}^{b+c} g$.

(C) $g \in R[a,b]$.

20. Which of the following is true ?

(A) Every convergent sequence is Cauchy.

(B) Every Cauchy sequence is convergent.

(C) Both (A) and (B).

21. Let $f : [a, b] \to \mathbb{R}$. Then which of the following is true ?

(A) f is continuous on $[a, b] \Rightarrow f \in R[a,b]$.

(B) $f \in R[a,b] \Rightarrow f$ is continuous on $[a,b]$.

(C) $f \in R[a,b] \Rightarrow f$ is uniformly continuous on $[a,b]$.

22. Let $f : [a, b] \to \mathbb{R}$. Then which of the following is true ?

(A) f is monotone on $[a,b] \Rightarrow f \in R[a,b]$.

(B) $f \in R[a,b] \Rightarrow f$ is monotone on $[a,b]$.

(C) None of these.

23. Let f be continuous on $[a, b]$ and $f(x) \geq 0$ for all $x \in [a, b]$. If $\int_a^b f = 0$, then which of the following is true ?

(A) $f(x) = 0$ for all $x \in [a, b]$.

(B) $f(x) = 0$ for all but finitely many points of $[a, b]$.

(C) None of these.

24. If f is bounded on $[a, b]$ and if 'f' restricted to $[c, b]$, $c \in (a, b)$ is Riemann integrable, then which of the following does not hold true ?

(A) $f \in R[a, b]$.

(B) $f \notin R[a, b]$.

(C) $\underset{c \to a^+}{\text{Lt}} \int_c^b f = \int_a^b f$.

25. Define 'g' in $[0, 1]$ as $g(x) = \begin{cases} \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$ Then which of these is true ?

(A) $g \in R[0, 1]$.

(B) $g \notin R[0, 1]$.

(C) None of these.

26. If 'f' is continuous on $[-1, 1]$ then $\int_0^{\pi/2} f(\cos x).\, dx$ is not equal to which of the following ?

(A) $\int_0^{\pi/2} f(\sin x).\, dx$.

(B) $\frac{1}{2} \int_0^{\pi} f(\sin x)\, dx$.

(C) $\int_0^{\pi} f(\sin x)\, dx$.

27. If 'f' is continuous at every point of $[a, b]$ and F is any antiderivative of 'f' on $[a, b]$, then $\int_a^b f(x) \cdot dx =$

(A) $(b-a)(F(b)-F(a))$.

(B) $F(b) - F(a)$.

(C) None of these.

28. If $g(x) = \begin{cases} x, & |x| \geq 1 \\ -x, & |x| < 1 \end{cases}$ and if $G(x) = \frac{1}{2}|x^2 - 1|$, then $\int_{-2}^{3} g(x)\, dx =$

(A) $\frac{1}{2}$.

(B) $\frac{5}{2}$.

(C) $\frac{7}{2}$.

29. Let $B(x) = \begin{cases} \frac{-1}{2}x^2, & x < 0 \\ \frac{1}{2}x^2, & x \geq 0. \end{cases}$ Then $\int_a^b |x|\, dx =$

(A) $(b-a)[B(b) - B(a)]$.

(B) $B(b) - B(a)$.

(C) None of these.

30. If $f : [0,1] \to \mathbb{R}$ is continuous and $\int_0^x f = \int_x^1 f$ for all $x \in [0, 1]$, then :

(A) $f(x) = 0, \forall x \in [0,1]$.

(B) $f \in R[0, 1]$.

(C) All the above.

C 20207 (Pages : 3) Name.....................................

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SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 09—REAL ANALYSIS

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define uniform continuity of a function.
2. State Weierstrass approximation theorem.
3. Find $\|P\|$ if $P = \{0, 2, 3, 4\}$ in a partition of $[0, 4]$.
4. Give an example for a function which is not Riemann integrable.
5. Define step function.
6. State Lebesgue integrability criterion.
7. Define uniform convergence of a series of functions.
8. $\lim_{n \to \infty} \frac{x^2 + nx}{n}$.
9. Write an example for an absolutely convergent improper integral.
10. Cauchy principal value of $\int_{-\infty}^{\infty} x\,dx =$.
11. Define Beta function.
12. Fill in the blanks : $\Gamma(3) =$ ———.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Let $I = [a, b]$ be a closed boudned interval and $f : I \to \mathbb{R}$ be continuous on I. If $k \in \mathbb{R}$ is any number satisfying $\inf f(I) \le k \le \sup f(I)$ then prove that there exists a number $c \in I$ such that $f(c) = k$.
14. State and prove Preservation of Intervals theorem.

Turn over

15. Show by an example that every uniformly continuous function need not be a Lipschitz function.

16. If $\phi:[a,b]\to\mathbb{R}$ is a step function, prove that $\phi\in\mathfrak{R}[a,b]$.

17. Suppose that $f,g\in\mathfrak{R}[a,b]$. Prove that $fg\in\mathfrak{R}[a,b]$.

18. State the substitution theorem of Riemann integration. Use it to evaluate $\int_0^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\,dt$.

19. Let (f_n) be a sequence of bounded functions of $A\subseteq\mathbb{R}$. Suppose that $\|f_n-f\|_A\to 0$. Then prove that (f_n) converges uniformy on A to f.

20. If f_n is continuous of $D\subseteq\mathbb{R}$ to $\mathbb{R}$ for each $n\in\mathbb{N}$ and if $\sum f_n$ converges to f uniformly on D, then prove that f is continuous on D.

21. State and prove Weierstrass M-Test for a series of functions.

22. Discuss the uniform convergence of $\sum_{n=1}^{\infty}\frac{1}{x^2+n^2}$.

23. Test the convergence of $\int_0^{\infty}\frac{1}{x^2}\,dx$.

24. Show that $\Gamma(n+1)=n!$ when n is a positive integer.

25. Show that $\beta(m,n)=\beta(n,m)$.

26. Evaluate $\int_0^1 x^m\left(\log\frac{1}{x}\right)^n dx$.

(10 × 4 = 40 marks)

Part C

Answer any* six *questions.
Each question carries* 7 *marks.

27. **Let $I=[a,b]$ be a closed bounded interval and let $f: I\to\mathbb{R}$ be continuous on I. Then prove that f is bounded on I.**

28. **State and prove Uniform Continuity Theorem.**

29. **State and prove Continuous Extension Theorem.**

30. **If $f:[a,b]\to\mathbb{R}$ is monotone on $[a,b]$ then prove that $f\in\mathfrak{R}[a,b]$.**

31. **Discuss the convergence of the sequence $(f_n(x))$ where $f_n(x)=\frac{x^n}{x^n+1}$, $x\in[0,2]$.**

32. **State and prove Taylor's Theorem with the Reminder.**

33. Let $f \in \mathfrak{R}[a,b]$ and let f be continuous at a point $c \in [a,b]$. Prove that the indefinite integral $F(z) = \int_a^z f$ foe $z \in [a,b]$ is differentiable at c and $F'(c) = f(c)$.

34. Show that $\Gamma\left(\frac{P}{2}\right)\Gamma\left(\frac{P+1}{2}\right) = \frac{\sqrt{\pi}}{2^{p-1}}\Gamma(p)$.

35. Evaluate the integral $\int_0^1 x^2\left(1-\sqrt{x}\right)dx$.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. (a) State and prove Maximum Minimum Theorem.

(b) Test the uniform continuity of $f(x) = \sqrt{x}$ on [0, 2].

37. (a) Let $f : [a,b] \rightarrow \mathbb{R}$ and $c \in (a,b)$. Prove that $f \in \mathfrak{R}[a,b]$ if and only if its restriction to [a, c] and [c, b] are both Riemann integrable. In this case show that $\int_a^b f = \int_a^c f + \int_c^b f$.

(b) If $f \in \mathfrak{R}[a,b]$ and if $[c,d] \subseteq [a,b]$ then prove that the restriction of f to [c, d] is in $\mathfrak{R}[c, d]$.

38. (a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}, \forall m, n > 0$.

(b) Find the value of $\Gamma\left(\frac{1}{2}\right)$.

(2 × 13 = 26 marks)