FIRST SEMESTER (CBCSS–UG) DEGREE EXAMINATION NOVEMBER 2021

B.Sc. Mathematics and Physics (Double Main)

MTS 1B 21—BASIC CALCULUS

(2020 Admissions)

Time : Two Hours and a Half   Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 25 marks.

1. Give the domain of the function $f(x) = \dfrac{x^2 - 1}{x - 1}$. Also find $f(0)$.

2. If $f(x) = \begin{cases} x & \text{if } x < 1 \\ 2 & \text{if } x = 1, \text{then find } \lim\limits_{x \to 1} f(x). \\ -x + 2 & \text{if } x > 1 \end{cases}$

3. Find the derivative with respect to x of $f(x) = x^3 - x$.

4. Find the local linear approximation of $f(x) = x^4$ at $x = 1$.

5. On what interval is $f(x) = x^3 - 2x + 6$ increasing or decreasing ?

6. $\int \csc x \cot x \, dx = $ _____.

7. Define horizontal asymptote.

8. If $f'(x) = 0$ for all x in an interval (a, b) then f is constant on (a, b).

9. Find the average value of $f(t) = \sin t$ on $[0, 2\pi]$.

10. If f is smooth on $[a, b]$, the length of the curve $y = f(x)$ from a to b is ————.

11. Find the work done in lifting a 1000 lb object 2 ft off the ground.

12. State Part 2 of the Fundamental Theorem of Calculus.

Turn over

13. Find the derivative of $y = \ln 75x$ with respect to x.

14. Find the inverse of the function $f(x) = \sqrt[3]{x-2}$.

15. Evaluate $\int\limits_{\ln 4}^{\ln 9} e^{x/2} dx$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 35 marks.

16. Find $\lim\limits_{x \to 0} \dfrac{\sqrt{x^3 + 100} - 10}{x^2}$.

17. Define $f(1)$ in a way that extends $f(s) = \dfrac{\left(s^3 - 1\right)}{\left(s^2 - 1\right)}$ to be continuous at $s = 1$.

18. Find the absolute extrema of $h(x) = x^{2/3}$ on $[-2, 3]$.

19. Show that the points of inflection of the curve $y^2 = (x-a)^2 (x-b)$ lie on the line $3x + a = 4b$.

20. Show that :

$$\int\limits_a^b c \, dx = c(b-a), c \text{ any constant.}$$

21. By integrating with respect to y, find the area between the graphs $x = y^2 - 2$ and $y = x$.

22. Find $\dfrac{dy}{dx}$, where $y = x \sinh x - \cosh x$.

23. Evaluate $\int\limits_1^{\infty} \dfrac{\ln x}{x^2} \, dx$, if it exists.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *marks.*

24. Differentiate the function $f(x) = \dfrac{8}{\sqrt{x-2}}$. Then find an equation of the tangent line at the point

    (6, 4) on the graph of the function.

25. Verify Rolle's Theorem for the function f defined by :

    $$f(x) = \frac{x^3}{3} - 3x, \; x \in [-3, 3].$$

26. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and

    the line $x = 3$ about the line $x = 3$.

27. Using L'Hôpital's Rule, evaluate $\displaystyle\lim_{x \to 2^+} \dfrac{x^2 + 3x - 10}{x^2 - 4x + 4}$.

    (2 × 10 = 20 marks)

(Pages : 2)

Name.....................................

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FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

B.Sc. Mathematics and Physics (Double Main)

PHY1B21—MECHANICS–I

(2020 Admissions)

Time : Two Hours

Maximum : 60 Marks

Ceiling for Section A is 20 marks and Section B is 30 marks.

Answer any one from Section C (10 marks).

Section A

Each question carries 2 marks.

1. 'Newton's first law of motion is the assertion that inertial systems exist". Justify this Statement.

2. Define "fictitious force". Give an example.

3. Derive the equation for period of a conical pendulum.

4. 2 astronauts are involved in a tug of war in free space. Draw the force diagram for the problem and write down the equation of force.

5. Compare gravitational and inertial masses.

6. State and explain work energy theorem in one dimension.

7. Show that electrostatic force is conservative.

8. Define the three cases of equilibria in terms of potential energy.

9. A particle is moving in x-z plane has mass m. Find its angular momentum.

10. Show that Angular momentum $L = dN/dt$, where N = torque.

11. State and explain parallel axes theorem.

12. Define radius of gyration. How is it related to moment of inertia ?

(Ceiling : 20 marks)

Section B

Each question carries 5 marks.

13. Two bodies of masses M_1 and M_2 ($M_1 > M_2$) are connected by a string of length l and passed over a frictionless pulley. If the pulley is moving upwards with an acceleration **A** find the relationship between accelerations of the masses and acceleration **A**.

14. A 5-kg mass moves under the influence of a force $F = (4t^2i - 3tj)$ N, where t is the time in seconds. It starts from the origin at $t = 0$. Find : (a) Its velocity ; (b) Its position ; and (c) $r \times v$, after 5 seconds.

15. A block of mass m rests on a fixed on a fixed wedge of angle θ. The co-efficient of friction is μ. Find the value of θ at which the block starts to slide.

16. Derive a general expression for the potential energy of inverse square law force.

17. A 900 kg. sports car accelerates to 100 kmph in 8 s. What is the average power that the engine delivers to the car's motion during this period ?

18. Calculate the moment of inertia of a uniform metre scale of mass 200 gm. about a perpendicular axis passing through 25 cm. mark.

19. A flat circular disc of mass 1 kg and diameter 2 m. rotates about an axis perpendicular to its plane and passing through its centre at 120 revolutions per minute. Find its kinetic energy and the torque required to stop it in 10 seconds.

(Ceiling : 30 marks)

Section C

Answer any **one** *question.*
The question carries 10 *marks.*

20. Give a brief account of everyday forces of physics. Give the important characteristics and mathematical outline of each force.

21. Explain a physical pendulum. Write down its equation of motion and hence arrive at its period of oscillation. Show that the expression gets reduced to that for simple pendulum for radius of gyration equal to zero ($k = 0$).

(1 × 10 = 10 marks)

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics and Physics (Double Main)

MTS 1B 21—BASIC CALCULUS

· (2021 Admissions)

Time : Two Hours and a Half   Maximum : 80 Marks

Section A

Answer atleast **ten** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall ceiling 30.

1. Find $\lim\limits_{t \to 0} \mathrm{H}(t)$ where $\mathrm{H}(t) = \begin{cases} 1 \text{ if } t \geq 0 \\ 0 \text{ if } t < 0 \end{cases}$.

2. Find a point on the graph of $f(x) = x^4 - 2x^2 + 2$ where the tangent at this point is horizontal.

3. Find the point at which the function $f(x) = |2x - 1|$ is not differentiable.

4. Find the marginal revenue funtion if the revenue function is :

    $\mathrm{R}(x) = 0.01x^3 - 0.001x^2 + 80x, 0 \leq x \leq 50$.

5. State Rolle's Theorem.

6. Define anti-derivative of a function.

7. Find a vertical asymptote of the graph of the function $f(x) = \dfrac{1}{x - 1}$.

8. Find the x-co-ordinate of the centroid of the region $y = \sqrt{x}$ in $[0, 4]$.

9. State Mean Value Theorem for integrals.

Turn over

10. Find $\displaystyle\lim_{x \to 1^{'}} \frac{\sin(\pi x)}{\sqrt{x-1}}$.

11. Find the work done by a force $F(x) = 3x^2 + x$ in moving a particle along the x-axis from $x = 2$ to $x = 4$.

12. Sketch the graph of the natural logarithmic function by defining it.

13. Evaluate $\int e^{-x} \sec\left(e^{-x}\right) dx$.

14. Find $\dfrac{dy}{dx}$ if $y = \dfrac{(2x-1)^3}{\sqrt{3x+1}}$.

15. Define jerk of a moving body on a co-ordinate line at a time t.

(10 × 3 = 30 marks)

Section B

Answer atleast **five** *questions.*
Each question carries 6 marks.
All questions can be attended.
Overall ceiling 30.

16. If we want the sum S of a positive real number x and its reciprocal to be as smallest as possible, find that number and the smallest sum S.

17. If $f'(x) = 0$ for all $x \in (a, b)$, show that f is constant in (a, b).

18. Determine the intervals where the graph of $f(x) = x^4 - 4x^3 + 12$ is concave upward and concave downward.

19. Find the slant asymptote of the graph of $f(x) = \dfrac{2x^3 - 3}{x - 2}$.

20. Find the volume of a sphere of radius r using integration.

21. A man has 100 feet of fencing to enclose a rectangular garden in his backyard. Find the dimension of the garden of the largest area he can have if he uses all of his fencing.

22. Show that $f(x) = x^3 + x + 1$ has exactly one solution or zero in the interval $[-2, 2]$.

23. The edge of a cube was measured and found to be 3 inches with a maximum possible error of 0.02. Find the appoximate maximum possible percentage error that would be incurred in computing the volume of the cube using this measurement.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Show that the function f defined by $f(x) = \sqrt{4 - x^2}$ is continuous through out the interval $[-2, 2]$.

   (b) Draw the graph of $g(x) = x^2 \sin \dfrac{1}{x}$ and find the interval or intervals where it is continuous.

25. The total annual revenue R of a company in thousands of dollars is related to the amount of money x that it spends on advertisements is given by $R = -0.01x^3 + 1.5x^2 + 200, 0 \le x \le 100$. Find the inflection points of R and interpret your findings.

26. State and prove Fermat's theorem for relative extremum.

27. (a) Find $\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x$ by writing the proof in detail.

   (b) Find $\sqrt{3.9}$ by method of linearization of $f(x) = \sqrt{x}$ at $x = 4$.

(2 × 10 = 20 marks)

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics and Physics (Double Main)

PHY 1B 21—MECHANICS–I

(2021 Admissions)

Time : Two Hours 	 Maximum : 60 Marks

Section A

*Answer at least **eight** questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. How are the standard units of length, mass and time defined ?

2. Draw the force diagram of a simple pendulum and hence arrive its equation of motion.

3. Graphically represent the gravitational force of a spherical shell and solid sphere.

4. What is the "turtle in elevator" problem ?

5. Distinguish between centre of mass and centre of gravity.

6. What is the curl of a vector ? When is it called irrotational ?

7. Explain how works is defined in mechanics. Show the geometrical representation of work done by a variable force.

8. Derive the equation for work energy theorem for vertical motion under gravity.

9. What is escape velocity ? What is its value for earth bound systems ?

10. Show that torque is zero in central force motion.

11. Derive moment of inertia in terms of angular momentum.

12. State the law of conservation of angular momentum. Give an example to prove it.

(8 × 3 = 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Two rectangular blocks of masses $m_1 = 2\,\text{Kg}$ and $m_2 = 1\,\text{Kg}$ are kept in contact on a smooth horizontal surface and pushed from behind with a force F = 2 N. Find the force of contact between them.

14. In a concrete mixer, cement, gravel, and water are mixed by tumbling action in a slowly rotating drum. Assume that the drum of a mixer has radius R and that it is mounted with its axle horizontal. What is the fastest the drum can rotate without the ingredients sticking to the wall all the time ?

15. The centre of mass of 3 particles of masses 1, 2 and 3 kg is at the point $\begin{pmatrix} 3 & 3 & 3 \end{pmatrix}$ where should a 4-th particle of mass 4 kg be placed so that the centre of mass be shifted to the point $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$.

16. Explain how the force F(r) can be found from the potential energy $U\left(r\right)$.

17. Show that curl of electrostatic field due to a charge $Q, F = K\,Q/r^2$ is zero where K is a constant.

18. State and prove parallel axes theorem.

19. Four 10 g masses are placed at the corners of a square of side .1 m. Find the moment of inertia of the arrangement about an axis passing through its centre perpendicular to its plane.

$(5 \times 5 = 25\ \text{marks})$

Section C

Answer any **one** *question.*
The question carries 11 marks.

20. State Newton's laws of motion. Explain their significance and the general procedure for the application of Newton's laws in solving mechanical problems. What are the limitations of Newtonian mechanics ?

21. Discuss the various ways of calculating the angular momentum of a particle moving in xy plane

$(1 \times 11 = 11\ \text{marks})$